

Technical Support Document C

Calibration and Validation Datasets for the Fox Illinois River Basin TMDL

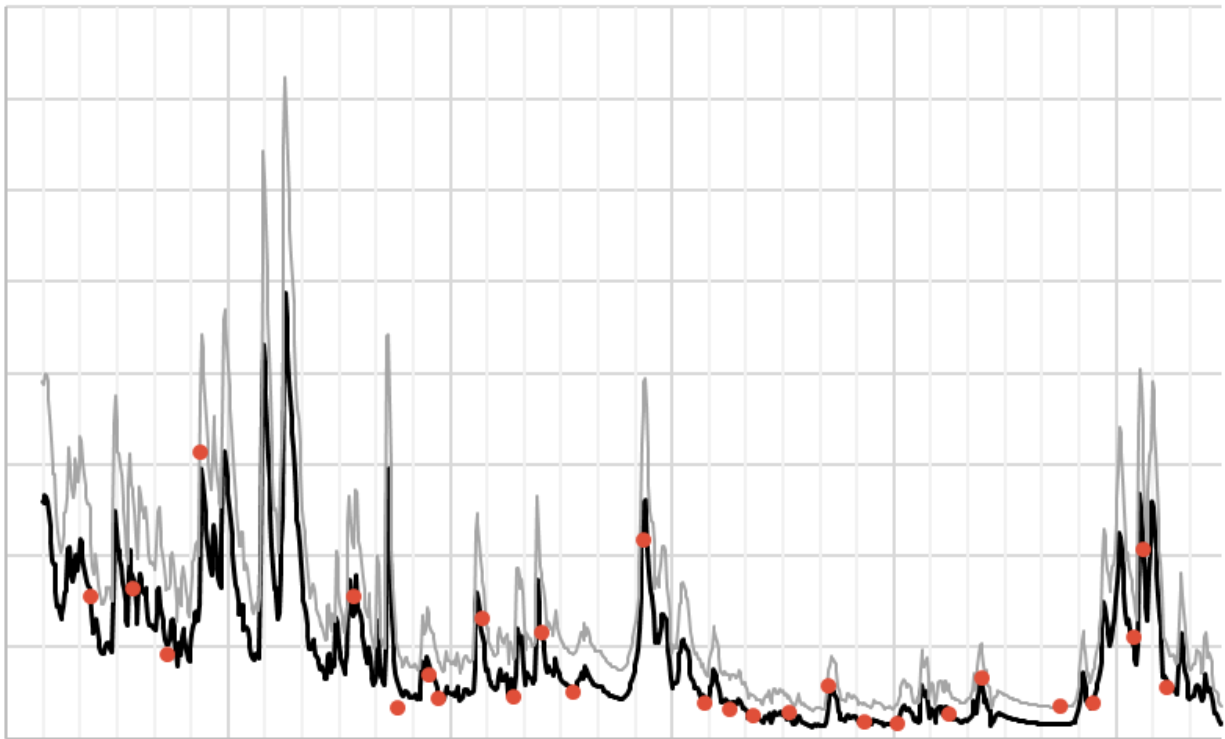
Supporting the Fox Illinois River Basin Total Maximum Daily Load (TMDL) for Phosphorus and Total Suspended Solids

Final
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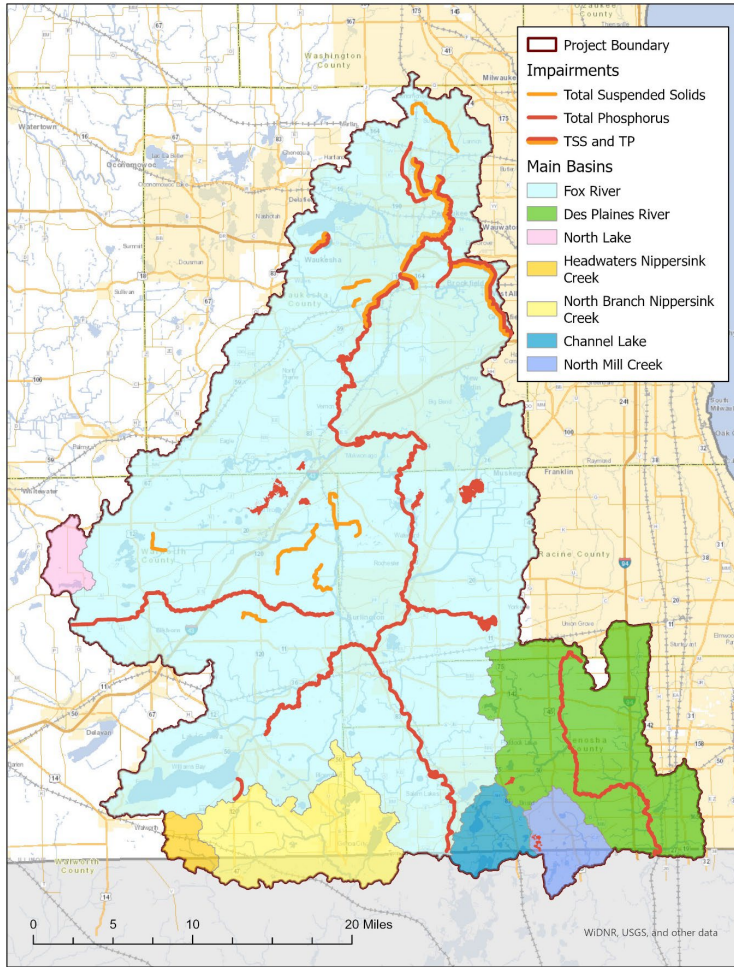
1. PROJECT BACKGROUND

The Department of Natural Resources (Department), together with many partners, is working to improve the surface water quality of tributaries, streams, rivers, and lakes within the Fox Illinois River Basin. To strengthen these ongoing efforts, the Department is developing a Total Maximum Daily Load (TMDL) for the river basin. The TMDL for this study area, referred to as the Fox Illinois River Basin (FOXIL) TMDL, will be a multi-year effort to address surface water quality impairments caused by phosphorus and total suspended solids. The TMDL study will provide a strategic framework and pollutant reduction goals for surface water quality improvement within the river basins.

The Fox Illinois River TMDL study area is located in southeastern Wisconsin. The study area includes the Fox River, the Des Plaines River, Nippersink Creek, North Mill Creek, and Channel Lake watersheds. The study area is primarily located in Racine, Kenosha, Walworth, and Waukesha counties. It is approximately bounded by Waukesha to the north, Lake Geneva to the southwest, and the western portions of Kenosha to the southeast. The TMDL study area covers approximately 1,060 square miles within Wisconsin, which is approximately 2 percent of the state. Within the study area, some lakes and streams are impaired (Wisconsin Department of Natural Resources, 2022), which means they are not meeting their water quality criteria. The extent of the TMDL and the waterbodies that are currently impaired are shown in Figure 1.1.

An important step in developing a TMDL is characterizing flows and loads at various locations in the watershed. The Department collected monitoring data in the project area from 2019 to 2022. These monitoring data were evaluated to develop estimates of continuous flow and estimates of daily load. The resulting datasets were used in the calibration and validation of a watershed model for the TMDL study area.

FIGURE 1.1
Extent of Fox Illinois TMDL Study Area



2. MONITORING SUMMARY

A monitoring plan for the Fox Illinois River Basin TMDL was implemented between December 2019 and May 2022. Monitoring was required to ensure adequate data were available for the calibration and validation of the watershed model. Water level, flow, and water chemistry data were collected at 13 sites during the monitoring period. A summary of the monitoring program is provided below, but a comprehensive report detailing the monitoring efforts are available in a separate report (Wisconsin Department of Natural Resources, 2023a).

2.1. Stage and Flow

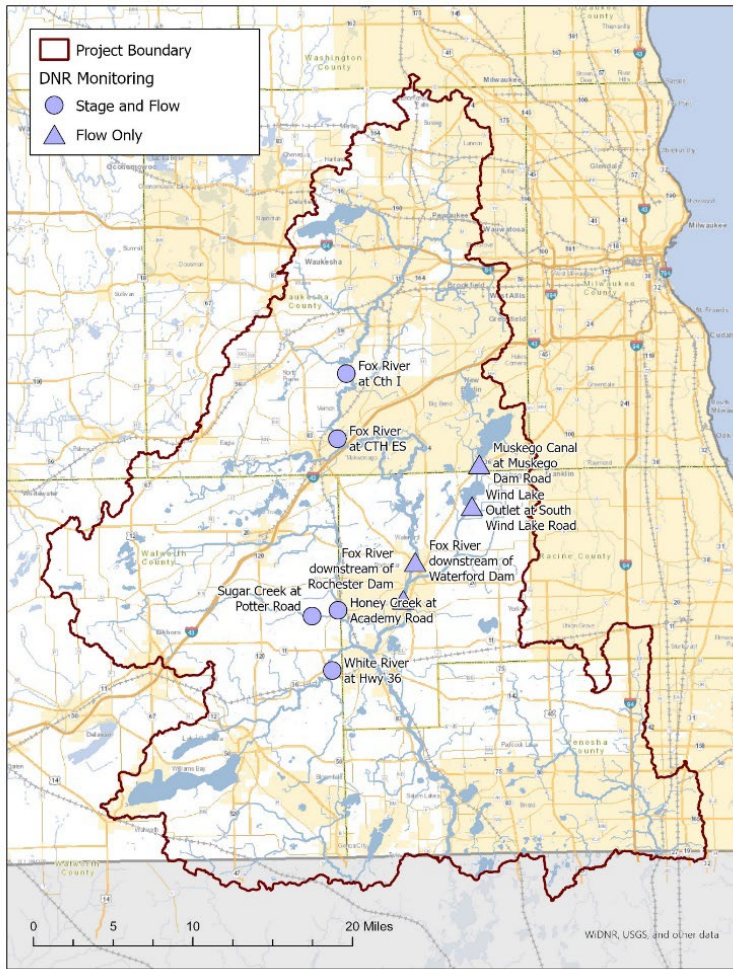
Stage and flow monitoring data were also collected during the monitoring period. The Department collected periodic flow measurements and continuous stage data at five sites and periodic flow data at four sites. The sites with only flow measurements were located near gages maintained by the United States Geological Survey that had stage data available. A summary of the stage and flow monitoring sites is provided in Table 2.2, and the location of each site is provided in Figure 2.2. Results from the stage and flow monitoring are available in a separate report (Wisconsin Department of Natural Resources, 2023a).

TABLE 2.1

Fox Illinois Rivers TMDL Stage and Flow Monitoring Sites

Stage and Flow Measurement Location	Stage data	Flow Data
Fox River at Cth I	DNR	DNR
Fox River at CTH ES	DNR	DNR
Honey Creek at Academy Road	DNR	DNR
Sugar Creek at Potter Road	DNR	DNR
White River at Hwy 36	DNR	DNR
Fox River downstream of Waterford Dam	USGS	DNR
Fox River downstream of Rochester Dam	USGS	DNR
Muskego Canal at Muskego Dam Road	USGS	DNR
Wind Lake Outlet at South Wind Lake Road	USGS	DNR

**FIGURE 2.1
Fox Illinois River TMDL Stage and Flow Monitoring Locations**



2.2. DNR Water Chemistry

Water chemistry data were collected at 13 locations in the study area. Water samples at five sites were collected by the Department, and samples at the remaining eight sites were collected by a private consultant, Cadmus. Water samples were evaluated for total phosphorus, orthophosphate, and total suspended solids. A list of the monitoring stations is provided in Table 2.1, and the locations of the stations are displayed in Figure 2.1. Results from the chemistry monitoring are available in a separate report (Wisconsin Department of Natural Resources, 2023a).

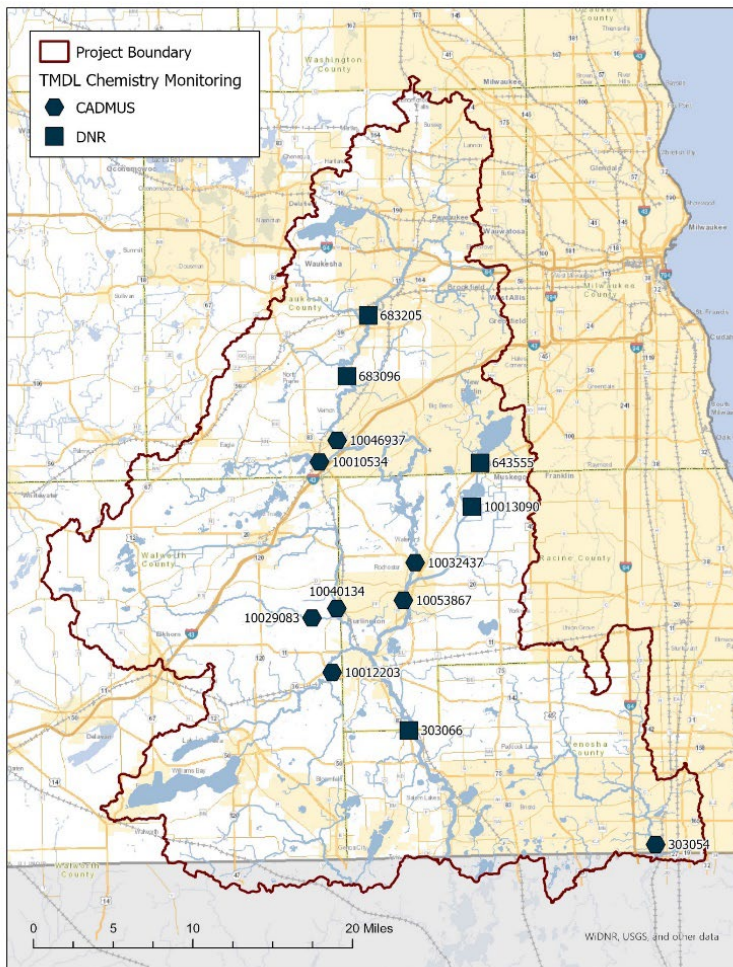
**TABLE 2.2
Fox Illinois River TMDL Chemistry Monitoring Sites**

SWIMS ID	SWIMS Station Name	Monitoring Entity	Chemistry Parameters
683205	Fox River - Ds Sunset Dr Bridge (Waukesha)	DNR	TP, TSS
683096	Fox River at Cth I Bridge	DNR	TP, TSS, TN, DOP, NO3, NH4
10046937	Fox River at CTH ES	Cadmus	TP, TSS, DOP

SWIMS ID	SWIMS Station Name	Monitoring Entity	Chemistry Parameters
303066	Fox River (II) - Nr New Munster Cthjb	DNR	TP, TSS, TN, DOP, NO3, NH4
10032437	Fox River at STH 20/30 Waterford	Cadmus	TP, TSS, DOP
10053867	Fox River at Case Eagle Park Bridge	Cadmus	TP, TSS, DOP
10010534	Mukwonago River (1) - Upstream of HWY 83	Cadmus	TP, TSS, DOP
643555	Muskego (Big Muskego) Lake - Outlet Near Wind Lake	DNR	TP, TSS
10013090	Wind Lake Canal_Wind Lake Upstream To Ceasars Dam	DNR	TP, TSS
10040134	Honey Creek at CTH DD/Academy Rd	Cadmus	TP, TSS, DOP
10029083	Sugar Creek at Potter Road	Cadmus	TP, TSS, DOP
10012203	White River - 10 M Upstream Of Hwy 36	Cadmus	TP, TSS, DOP
303054	Des Plaines River at Cth ML	Cadmus	TP, TSS, DOP

Parameters: DOP = Dissolved orthophosphate, NH4 = Ammonium, NH3 = Nitrate, TN = Total Nitrogen, TP = Total Phosphorus, TSS = Total Suspended Solids

FIGURE 2.2
Fox Illinois River TMDL Chemistry Monitoring Locations



2.3. Supplemental Water Quality Data

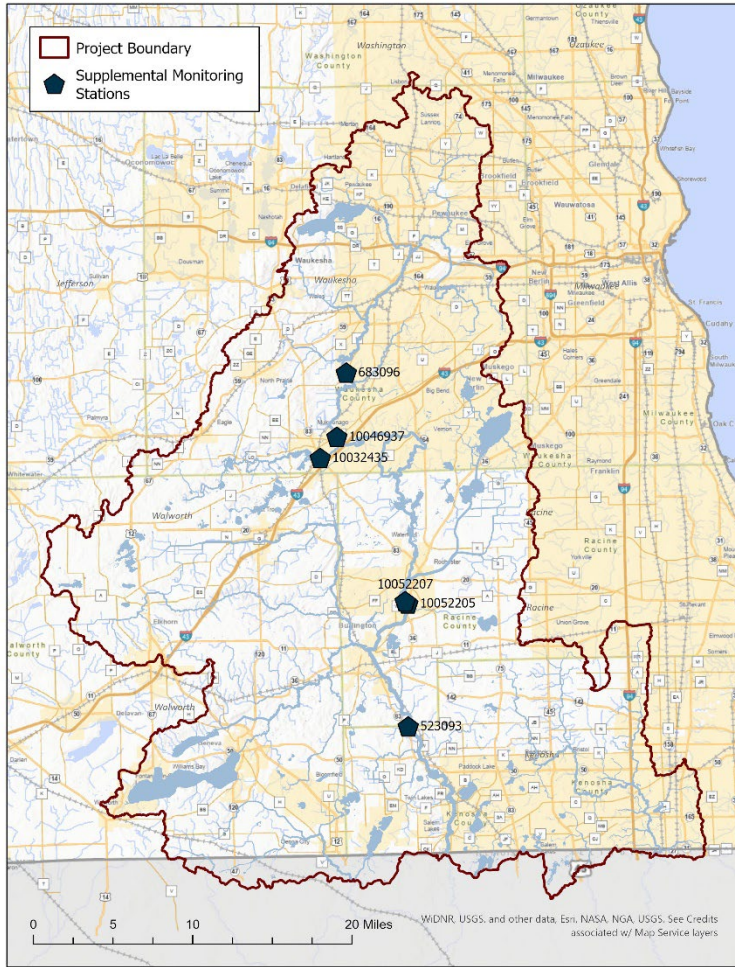
Additional water quality data in the study area has been collected by various entities. Only water quality data collected near sites with continuous flow data were used for calculating loads. Load estimation process is explained in Section 4. A list of stations with applicable water quality data load estimation Table 2.3. A map of the locations with applicable water quality data is provided in Figure 2.3.

TABLE 2.3

Supplemental Water Quality Stations Used for Load Estimation

SWIMS ID	SWIMS Station Name	Monitoring Entity	Corresponding Flow Station
10032435	Mukwonago River at STH 83 bridge	Mukwonago WWTF	Mukwonago River
683096	Fox River at Cth I Bridge	Mukwonago WWTF	Fox River at Cth I
10046937	Fox River at CTH ES	Mukwonago WWTF	Fox River at Cth ES
10052205	Fox River at STH 83 Rochester	WRCSD	Fox River at Rochester
10052207	Fox River WRCSD 430 M US STH 83	WRCSD	Fox River at Rochester
523093	Fox River at Cth Jb	Burlington WWTP	Fox River near New Munster

FIGURE 2.3
Fox Illinois River TMDL Supplemental Chemistry Monitoring Stations



3. CONTINUOUS FLOW ESTIMATION

Calibration of the watershed model developed for the FOXIL TMDL required continuous flow estimates. The stage and flow data summarized in the previous section were used to develop the continuous flow measurements. Rating curves—which relate flows to stage—were established at six sites in the study area. Sufficient stage data were not available at three sites, so an alternative approach was used that utilized linear relationships with flow measurements at nearby USGS gage stations (an enhancement to the area-weighted approach that can be used to estimate flows at different locations in a stream). The following sections describe the two methods for developing continuous flow estimates.

3.1. Stage-Discharge Relationships

One approach for producing estimates of continuous flow is the development of a stage-discharge relationships at each monitoring site. The stage-discharge relationship is expressed as an equation that estimates flow rate from stage. At six locations sufficient stage and discharge data were available to develop these relationships, and the methods are described in the following sections.

3.1.1. Development of Stage-Discharge Relationships

Continuous flow estimates were generated by developing stage-discharge relationships that relate periodic flow measurements to continuous stage measurements. Equation 3.1 provides the general form of the stage-discharge relationship (Hamilton, Watson, & Pike, 2019):

$$Q = C_0(H - e)^B \quad \text{Equation 3.1}$$

where:

- Q: Discharge
- C₀: Shape coefficient
- H: Stage
- e: Offset
- B: Shape exponent

The parameters in Equation 3.1 are linked to the physical features of a channel (Hamilton, Watson, & Pike, 2019). The coefficient (C₀) is the product of a coefficient describing the stage-area relationship and the flow resistance factor. The offset (e) is related to the stage at which flow is expected to be zero. The exponent (B) is the sum of a shape exponent in the stage-area relationship and a friction loss assumption. Since all of the parameters can be related to physical features, initial estimates of the coefficients can be derived based on the shape of the channel. Figure 3.1 is a table reproduced from Hamilton et al. (2019) that shows demonstrates the impact of channel shape on the stage-discharge equation.

Stage data were estimated from pressure transducers. The methods to convert pressure transducer data to stage data are described in the monitoring report for the FOXIL TMDL (Wisconsin Department of Natural Resources, 2023a). Sufficient continuous stage and discharge measurements were available at six monitoring locations. Table 3.1 lists the six stations and the source of the flow and stage measurements.

The channel shape at each of the locations with the flow data were estimated from data collected during the flow measurements. Flow measurements were collected with an Acoustic Doppler Current Profiler (ADCP), which provides data about depths along the width of the stream being measured. A channel shape at each of the sites were estimated from data collected on the day with the highest measured flow rate. The shapes of the cross-sections were used to ensure the exponent calculated for the stage-discharge relationship were physically reasonable. The cross-sections for each of the streams are provided in Figure 3.2.

FIGURE 3.1
Link Between Channel Shape and Rating Curve Equations
 Reproduced from Hamilton et al. (2019)







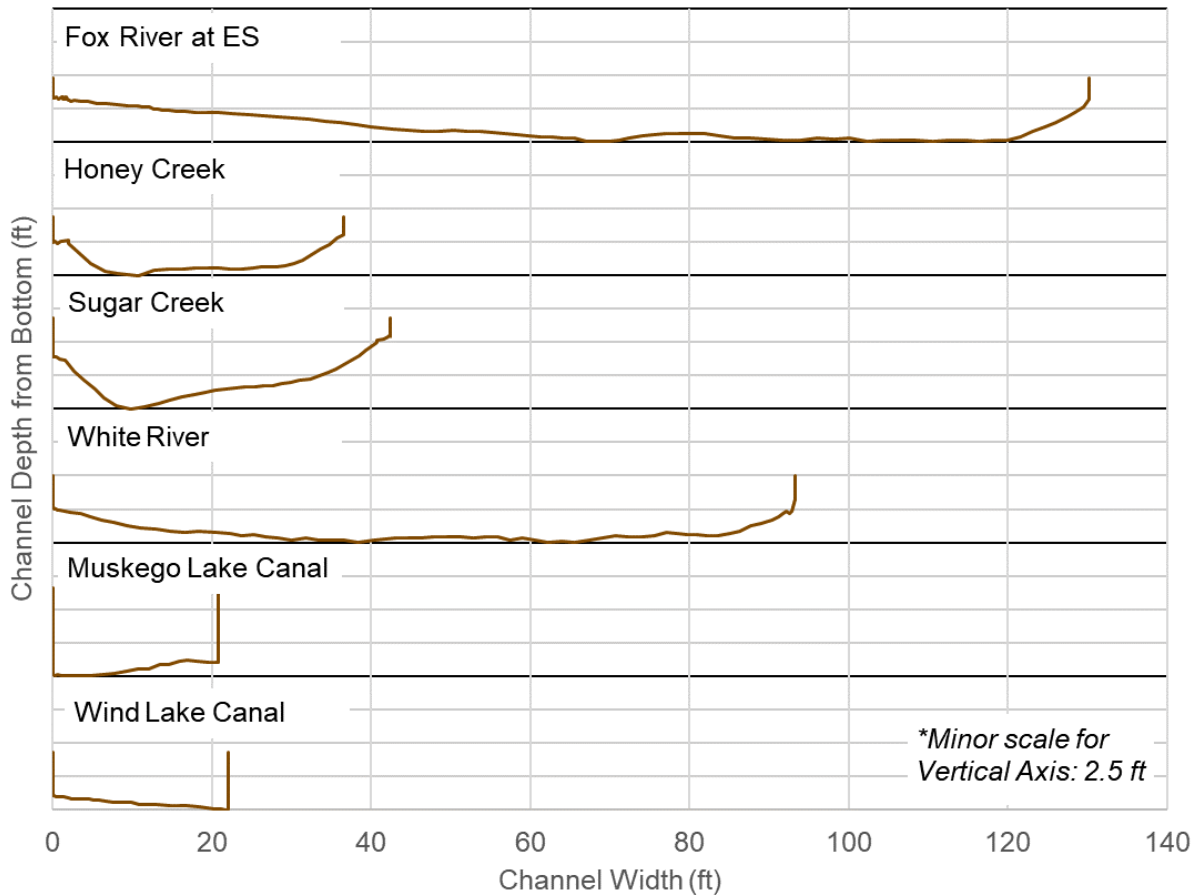
Control Shape	Stage-Radius $R=C_3(H-e)$	Stage-Area $A=C_4(H-e)^b$	Radius-Velocity $V=C_2R^x$	Stage-Discharge $Q = C_2C_3^xC_4(H-e)^{(b+xc)}$
	$C_3 = 1$	$b = 1$	$x = 0.67$	$Q=C_0(H-e)^{1.67}$
	$C_3 = 0.67$	$b = 1.25$	$x = 0.67$	$Q=C_0(H-e)^{1.92}$
	$C_3 = 0.63$	$b = 1.5$	$x = 0.67$	$Q=C_0(H-e)^{2.17}$
	$C_3 = 0.67$	$b = 1.5$	$x = 0.67$	$Q=C_0(H-e)^{2.17}$
	$C_3 = 0.54$	$b = 1.75$	$x = 0.67$	$Q=C_0(H-e)^{2.42}$
	$C_3 = 0.5$	$b = 2.0$	$x = 0.67$	$Q=C_0(H-e)^{2.67}$

TABLE 3.1
Monitoring Locations Used for Stage-Discharge Relationships

Location	Source of Stage Data	Source of Flow Data
Fox River at Cth ES	DNR	DNR
Honey Creek at Academy Road	DNR	DNR
Sugar Creek at Potter Road	DNR	DNR
White River at Hwy 36	DNR	DNR
Muskego Canal at Muskego Dam Road	USGS (05544385)	DNR
Wind Lake Outlet at South Wind Lake Road	USGS (424848088083100)	DNR

FIGURE 3.2
Cross-Sectional Areas at Measurement Locations



Rating curves at each monitoring location were constructed in six steps:

1. *Identification of breakpoints:* A breakpoint is defined as a depth within the stream at which the control—in this case the channel cross section—encounters a notable change. For example, the shape of the cross-section at Honey Creek (Figure 3.2) changed from a parabolic shape to a shape with near-vertical slopes at approximately 3 feet above the bottom of the channel. Since the rating curve equation was based on physical characteristics of the stream, the rating curve equation was likely different for depths below and above the breakpoint depth.
2. *Review of data for outliers:* Once breakpoints were established, datasets were reviewed for any potential outliers. Outliers were removed from the dataset for measurements where field staff indicated difficulties with the flow measurements collected.
3. *Transformation of data to log-log space:* Stage and discharge measurements for the remaining datapoints were converted to the natural log, which was based on recommendations from Department guidelines for developing rating curves (Wisconsin Department of Natural Resources, 2018).
4. *Calculation of linear regression statistics for log-transformed stage-discharge relationships:* The transformed dataset followed a linear trend, so statistics to determine linear fit were calculated. Linear regression statistics were calculated using the LINEST function in

Microsoft Excel (Microsoft Corporation, 2023) with the x values equal to the natural log of stage minus offset and the y values equal to the natural log of discharge.

5. *Adjustment of offset value to maximize coefficient of determination (R^2):* The Solver tool in Excel was used to iterate on the value of the offset to maximize the coefficient of determination. Coefficient of determination indicates the strength of a linear relationship, and a value closer to one indicates a strong linear relationship.
6. *Review of stage-discharge parameters:* The values calculated from the Solver equation were reviewed to ensure they were reasonable. First, the offset was examined to ensure the estimated value represented a stage relatively close to the expected depth of zero-flow. Second, the exponent was compared to the stream cross-sections and the cross-sections in Figure 3.1 to ensure the values were reasonable.

The results of the rating curve estimates are summarized in Table 3.2. The rating curves developed for the six sites and the observations are provided in Appendix A. Overall, the relationship of the rating curves was strong for Honey Creek, Sugar Creek, the White River, and the higher flows of Muskego Lake Canal. The discharges predicted by the stage-discharge relationship for the Wind Lake outlet are close to the observed discharges; however, the rating curve overpredicts flow for the highest flow measured. Wind Lake Canal may be influenced by a backwater effect from the Fox River near Rochester, which may explain why the measured flow is lower than the predicted flow. Without additional data, however, it is not possible to verify the rating curve for higher flows. Thus, continuous flow estimates from at the Wind Lake outlet must be used with caution.

TABLE 3.2
Parameters for Rating Curve Equations

Station	Curve 1 (Stage ≤ Breakpoint)				Breakpoint (ft)	Curve 2 (Stage > Breakpoint)			
	C_0 (cfs)	e (ft)	B	R^2		C_0 (cfs)	e (ft)	B	R^2
Fox River at ES	35.1	4.76	1.90	0.88	None				
Honey Creek	20.8	7.25	2.38	0.90	9.00	28.9	7.14	1.64	1.00
Sugar Creek	29.8	7.33	1.76	0.96	None				
White River	33.1	0.50	2.10	0.94	2.50	48.6	0.50	1.53	0.99
Wind Lake Outlet	93.3	4.50	1.53	0.76	None				
Muskego Lake Canal	19.2	8.50	1.84	0.61	8.64	19.2	7.50	1.90	0.91

3.1.2. Adjustments for Muskego Lake and Wind Lake

During the monitoring period, periods of zero flow were observed in the Muskego Lake channel and at the Wind Lake outlet. Outflow from Wind Lake and Muskego Lake was actively managed during the monitoring period to ensure adequate levels in the lakes were maintained. During dry periods the crests of the respective dams were above the water level in the lakes and no flow was discharged downstream.

Since only periodic flow measurements were taken during the course of the monitoring period, periods of zero flow had to be estimated when developing the continuous flow estimates. The USGS gages at both Muskego Lake and Wind Lake contain stage data within the lake and in the downstream channel. The stage data in the channel downstream of the lake was used to develop the

rating curves, but the stage data from the lake itself was useful for predicting periods of zero flow. To determine zero-flow periods, measured flows were plotted against the stage in the lake. From these plots, the approximate lake level maintained during different seasons was estimated. When measured lake level was less than the estimated maintained lake level, flow downstream was assumed to be zero. When measured lake level was greater than the estimated maintained lake level, the rating curve and the downstream stage data were used to estimate flows. A summary of the estimated maintained lake levels is provided in Table 3.3.

TABLE 3.3
Estimated Lake Level Maintained for Muskego and Wind Lakes

Location	Date Range	Maintained Lake Level ¹ (ft)
Muskego Lake	April 1 – October 31	11.52
	November 1 – March 31	11.75
Wind Lake	March 15 – October 14	8.2
	October 15 – March 14	7.6

1. When lake level measured at USGS gage is less than estimated maintained lake level, flow downstream is zero

3.1.3. Impacts of Ice on Flow Estimates

During the monitoring period, the streams being monitored were occasionally covered with ice. The presence of ice prevented flow data being collected. Ice also had an impact on stage measurements. Flows during periods when ice was present ('ice-on') could not be reliably predicted from the stage-discharge relationships, so no flow estimates are provided for these dates. These 'ice-on' periods were identified by reviewing the field data provided by the team conducting monitoring and by reviewing the stage data from the continuous level logger equipment.

3.1.4. Development of Continuous Flow Dataset

The rating curves and the information about lake level management were combined with measured continuous stage data to create continuous flow datasets. The estimated continuous flows compared to the measured flows are provided in Appendix A. Time periods with no flow estimates are also shown in the figures. Periods without flow estimates generally align with dates when ice was present. Dates without estimates outside of expected ice-on periods correspond to times where stage estimates from the level loggers were unavailable or unreliable. Details about the operation of the monitoring equipment is provided in the report detailing the monitoring activities for the TMDL (Wisconsin Department of Natural Resources, 2023a).

3.2. FLOW ESTIMATION USING LINEAR REGRESSION WITH USGS GAGES

Continuous flow estimates at three sites were developed by comparing periodically measured flows with continuous flows at nearby USGS gages. A linear regression relationship was fit to these data to develop continuous flow estimates at the stations with only periodic flow measurements. The monitoring locations utilizing linear regression with USGS flow data are summarized in Table 3.4

TABLE 3.4

Monitoring Locations Utilizing Linear Regression with USGS Stations

Location	USGS Gage Used for Relationship
Fox River at Cth I	USGS 05543830 – Fox River at Waukesha, WI
Fox River at Waterford	USGS 05545750 – Fox River near New Munster, WI
Fox River at Rochester	USGS 05545750 – Fox River near New Munster, WI

The linear regression method was used at these three locations because measured stage data prevented the development of reliable and robust stage-discharge relationships. At the Fox River at County Highway I, issues with the deployment of the level logger limited the length of the continuous stage records. Details about the challenges are provided in the monitoring report for this project (Wisconsin Department of Natural Resources, 2023a). Additionally, the Fox River at County Highway I is the location of a long-term trends water quality monitoring site operated by the Department. The long-term trends site contains a long-term dataset for water quality parameters. To best utilize these data, a long-term, continuous flow dataset was required to develop long-term load estimates. The load estimation procedure is described in detail in Section 4.

Stage data for the locations of the Fox River at Waterford and Rochester were available from the USGS; however, the stage gages are located upstream of the dams at Waterford and Rochester. The operation of the dams varies throughout the year, so stage alone is not a reliable predictor of flow. Due to these limitations, an accurate stage-discharge relationship could not be developed.

To estimate continuous flows at the three locations described above, periodic flow measurements were plotted against the flow measurements collected at the USGS gages on the same date. A linear regression relationship was fit to the plotted measurements to characterize the relationship between flows at the two locations. The equation from the linear regression is summarized in Equation 3.2. A summary of the linear regression relationships is provided in Table 3.5. A plot of the relationships are provided in Appendix B.

$$Q_{sta} = slp \times Q_{USGS} + intercept \quad \text{Equation 3.2}$$

where:

Q_{sta} : Discharge at the monitoring location

slp: Slope of regression relationship

Q_{USGS} : Discharge at the USGS gage

intercept: Intercept of the regression relationship

TABLE 3.5

Results of Linear Regression Equations for Estimating Flows

Monitoring Location	USGS Gage	Slope	Intercept	R ²
Fox River at Cth I	05543830	1.371	21.3	0.97
Fox River at Waterford	05545750	0.426	42.5	0.93
Fox River at Rochester	05545750	0.685	-40.7	0.93

The linear regression method is an enhancement to the area-weighted approach that can be used to estimate flows at different locations in a stream. In the area-weighted approach, a ratio of the

upstream areas from a location with flow measurements and a location with no flow measurements is calculated. This ratio is multiplied by flows at the location with measured flows to estimate flows at the location without measured flows. The linear regression approach was used instead of the area-weighted method because using the available measured flow data provided a more accurate representation of estimated flows.

The results of the linear regression analysis defined by Equation 3.2 and summarized in Table 3.5 were applied to the continuous flow records at the respective USGS stations to develop a continuous flow estimate for each of the monitoring locations. The resulting continuous flow estimates and the observed flows are provided in Appendix B.

4. DAILY FLUX (LOAD) ESTIMATION

Continuous flow measurements and periodic water quality monitoring data were used to establish daily flux estimates for fourteen locations in the study area. Daily flux for thirteen stations was estimated using a modified version of the Fluxmaster and LOADEST methods developed by USGS (Schwarz, Hoos, Alexander, & Smith, 2006). The technique used to estimate daily flux was also used for previous TMDLs developed by the Department. This section of the report describes the methods used for the load estimation. The following text is reproduced from Appendix J of the Department's Northeast Lakeshore TMDL report (Wisconsin Department of Natural Resources, 2023b).

4.1. Site Specific Flux Models

Continuous daily fluxes were estimated for both TP and TSS at each site in the monitoring network. Flux computation was performed with a modified version of the methods that are associated with U.S. Geological Survey Fluxmaster and LOADEST software programs (Schwarz, Hoos, Alexander, & Smith, 2006). The purpose of these methods is to estimate constituent concentrations at a given site when water quality sampling frequency is insufficient for estimating continuous long-term flux. The methods are most effective for constituents that have a strong relationship with discharge and exhibit cyclic variation with season (e.g., sediment concentration is often greatest with snowmelt events in late Spring). Additionally, a time variable allows concentrations to vary, linearly or quadratically, over the sampling period.

4.2. Modifications to LOADEST Model

The first purpose of modifying the Fluxmaster/LOADEST method was to rectify issues with marginal sample sizes for most sites in the monitoring network. Since the development of these tools, new statistical methods have become available that allow model coefficients to vary by a grouping factor (e.g., a monitoring site). Using each monitoring site as a grouping factor, a single model per constituent can be fitted without the loss of degrees of freedom that would result from multiple independent models for each site using the regression methods implemented in Fluxmaster/LOADEST. The modeling framework chosen for model fitting was an implementation of linear mixed effects models in the R programming language for statistical computing (R Core Team, 2020) the library used in R software was the lme4 package (Bates, Maechler, Bolker, & Walker, 2015). Most of the same methods were used by adapting computer code from the rloadest R package (Runkel & De Cicco, 2017), except when model fitting was performed using lme4.

The second purpose of modifying the Fluxmaster/LOADEST method was based on initial findings that quickflow (the combination of surface runoff and shallow aquifer water yield) was a better predictor of TP and TSS than total discharge. To calculate quickflow for each site, a baseflow separation routine was applied based on wavelet transform (Nathan & McMahon, 1990) that is available in an R package called EcoHydRology (Fuka, Walter, Archibald, Steenhuis, & Easton, 2015). The recommended parameters (filter parameter = 0.925, passes = 3) were used. In model fitting for both TSS and TP, quickflow was always used instead of total discharge. All quickflow values were translated by a value of positive 0.01 to prevent the log transformation of zero values during dry periods when baseflow accounted for all flow. Constituent concentration models were fitted using quickflow, however flux estimates were calculated by multiplying concentration predictions by total discharge.

4.3. Model Selection

The rloadest R package provides a convenience function that fits 9 different models that are different permutations of discharge, season, and time as fixed-effect co-variates, then selects the

best performing model as the one with the lowest AIC. Structuring these models as mixed-effect models using the lmer package in R allows more permutations of model coefficients. A mixed-effect model allows coefficients to vary by grouping factor, usually referred to as a “random effect”. In a mixed-effects model, coefficients that do not vary by grouping factor are referred to as “fixed effects”. Therefore, the 9 different models used by the rloadest package can be expanded to 30 with different permutations of discharge (i.e., quickflow), season, and time as both fixed effects and random effects. In cases where random effects were fitted, the coefficients were allowed to vary for each monitoring site. The intercept of all model permutations was allowed to vary by monitoring site.

Two models (one for TSS, and one for TP) were selected that predicted constituent concentrations across all monitoring sites. These models were selected by permuting through all combinations of fixed and random effects for flow, season, and time, then selecting those with the minimum AIC. The models selected for TSS (model 6c) is described by Equation 4.1, and the model selected for TP (model m9d) is described by Equation 4.2.

$$\ln(\boldsymbol{\rho}_m) = \beta_0 + [\beta_f] * \begin{bmatrix} \ln(Q_m) \\ \ln(Q_m^2) \\ \sin(2\pi T_m) \\ \cos(2\pi T_m) \end{bmatrix} + \gamma_0 + [\gamma_{f,m}] * \begin{bmatrix} \ln(Q_m) \\ \ln(Q_m^2) \\ \sin(2\pi T_m) \\ \cos(2\pi T_m) \end{bmatrix} + e_m \quad \text{Equation 4.1}$$

$$\ln(\boldsymbol{\rho}_m) = \beta_0 + [\beta_f] * \begin{bmatrix} \ln(Q_m) \\ \ln(Q_m^2) \\ \sin(2\pi T_m) \\ \cos(2\pi T_m) \\ T_m \\ T_m^2 \end{bmatrix} + \gamma_0 + [\gamma_{f,m}] * \begin{bmatrix} \ln(Q_m) \\ \ln(Q_m^2) \\ \sin(2\pi T_m) \\ \cos(2\pi T_m) \end{bmatrix} + e_m \quad \text{Equation 4.2}$$

In the above equation, m is a monitoring site, $\boldsymbol{\rho}$ is a matrix of TSS or TP concentrations, \boldsymbol{Q} is a matrix of flow paired with $\boldsymbol{\rho}$, \boldsymbol{T} is a matrix of decimal time numbers paired with $\boldsymbol{\rho}$, β is a fixed-effect coefficient, γ is a random-effect coefficient, and e is residual error.

Daily flux estimates were only generated for TSS at ten of the fourteen sites. Four of the sites with TSS data were located downstream of dams that impact the release of TSS. The models described above were tested using all fourteen sites, but an accurate model was not able to be established for the four sites downstream of dams. Despite the limitations with TSS data, accurate daily flux estimates were able to be established for TP at thirteen sites. Coefficients for both fixed and random effects are listed in Table 4.1 for TSS and Table 4.2 for TP.

TP estimates were established at only thirteen of the fourteen sites because data from the Des Plaines River at Cth ML was not included in the analysis for TP. The Des Plaines River watershed includes a significant amount of tile drainage, which affects the delivery of phosphorus. Since the phosphorus dynamics in the Des Plaines River is markedly different than the other thirteen sites, TP load estimates for the Des Plaines River were determined separately. The methods for developing load estimates for the Des Plaines River are described in Section 4.6.

TABLE 4.1

Coefficients for TSS Linear Mixed Effects Model

Monitoring Location	Intercept	$\ln(Q)$	$\ln(Q^2)$	$\cos(2\pi T)$	$\sin(2\pi T)$
Fixed Effects	2.08	0.0500	0.0090	-0.500	-0.011
Des Plaines River at Russell, IL	3.54	-0.0119	-0.0248	-0.831	-0.349
Fox River at ES	2.69	0.0242	-0.0073	-0.128	0.173
Fox River at I	2.33	0.0393	-0.0006	-0.093	0.216
Fox River at Rochester	1.75	0.0641	0.0143	-0.722	-0.146
Fox River at Waterford	1.90	0.0576	0.0076	-0.804	-0.232
Fox River at Waukesha	1.42	0.0780	0.0100	0.174	0.402
Fox River near New Munster	1.92	0.0568	0.0193	-0.795	-0.152
Honey Creek	1.81	0.0614	0.0188	-0.448	0.068
Sugar Creek	1.36	0.0807	0.0337	-0.647	0.000

TABLE 4.2

Coefficients for TP Linear Mixed Effects Model

Monitoring Location	Intercept	$\ln(Q)$	$\ln(Q^2)$	$\cos(2\pi T)$	$\sin(2\pi T)$	T	T ²
Fixed Effects	-2.76	-0.286	0.060	-0.355	-0.123	-0.0116	0.0078
Fox River at ES	-2.38	-0.297	0.074	-0.430	-0.167		
Fox River at I	-2.45	0.058	-0.010	-0.225	-0.147		
Fox River at Rochester	-2.50	-0.350	0.084	-0.486	-0.136		
Fox River at Waterford	-2.82	-0.126	0.035	-0.710	-0.241		
Fox River at Waukesha	-2.31	-0.084	0.024	-0.243	-0.147		
Fox River near New Munster	-2.34	-0.477	0.117	-0.512	-0.171		
Honey Creek	-2.59	-0.499	0.114	-0.467	-0.129		
Mukwonago River at Mukwonago	-3.45	-0.480	0.075	-0.054	0.050		
Muskego Lake	-3.27	-0.051	-0.002	-0.413	-0.128		
Sugar Creek	-2.30	-0.765	0.179	-0.465	-0.145		
White River	-2.59	-0.350	0.081	-0.454	-0.177		
White River at Lake Geneva	-3.64	-0.260	0.021	0.019	-0.001		
Wind Lake	-3.28	-0.039	-0.012	-0.176	-0.062		

4.4. Flux Estimation

To estimate flux for a given day, the estimated constituent concentrations from Equation 4.1 must first be re-transformed from natural log to real space then multiplied by the average daily discharge for each day. In the process of transforming back to real space, systemic biases in the predictions can occur due to heteroscedasticity in the linear model. A common approach to reduce bias in estimates is to multiply each concentration prediction by a bias-correction factor (BCF). The BCF that

is used in the LOADEST model is a function of the residual standard error (SE) of the mode (Runkel, Crawford, & Cohn, 2004)

$$BCF = \exp\left(\frac{SE^2}{2}\right) \quad \text{Equation 4.2}$$

Residual error varies for each monitoring site, and therefore site-specific biases can be resolved by calculation a BCF for each monitoring site, m .

$$BCF_m = \exp\left(\frac{SE_m^2}{2}\right) \quad \text{Equation 4.3}$$

The benefit of using a mixed-effect model is that coefficients are allowed to vary by grouping factors without the same reduction in degrees of freedom that would result in independent regression models fit for each group. However, there is some disagreement among statisticians about the interpretation of degrees of freedom associated with each group fit in a mixed-effects model. Therefore, the denominator in the equation used to calculate the SE of the residuals for each monitoring site, m , is subject to interpretation:

$$SE = \sqrt{\frac{\sum_i e_{i,m}^2}{d.f.}} \quad \text{Equation 4.4}$$

Multiple calculations for degrees of freedom for each monitoring site were tested, including sample size, sample size minus the number of fixed effects, and sample size minus the number of the total of fixed and random effects. Simply using the sample size as the degrees of freedom for each monitoring site provided an appropriate balance of bias correction across sites. For those monitoring sites where the number of samples are limited, the flux predictions should be used with caution, paying closest attention to the overall bias of predictions.

4.5. Assessment of Fit

For each constituent—TSS and TP—a single mixed-effect model was fitted that included samples across all monitoring sites. However, the flux estimates and associated characterization of model fit were used site-by-site. Therefore, all observed (sample concentration multiplied by mean daily flow on the date the sample was taken) and simulated fluxes were first separated out by site before calculating performance statistics (i.e., error is characterized as e_m from Equation 4.1). Performance statistics for TSS and TP for each site are shown in Table 4.3 and Table 4.4.

TABLE 4.3
Fit Statistics for TSS Mixed-Effects Model

Monitoring Location	n	BCF	PBIAS	NSE	R ²
Des Plaines River at Russell, IL	38	1.25	-4.8	0.28	0.46
Fox River at ES	26	1.18	12.6	0.55	0.71
Fox River at I	125	1.26	-5.7	0.28	0.43
Fox River at Rochester	38	1.18	5.6	0.20	0.74
Fox River at Waterford	86	1.39	-27.1	0.29	0.37
Fox River at Waukesha	40	1.28	-9.2	0.63	0.66
Fox River near New Munster	96	1.21	-14.5	0.52	0.53
Honey Creek	34	1.20	1.1	0.63	0.66
Sugar Creek	36	1.21	6.2	0.94	0.95

TABLE 4.4
Fit Statistics for TP Mixed-Effects Model

Monitoring Location	N	BCF	PBIAS	NSE	R ²
Fox River at ES	26	1.08	-0.5	0.80	0.82
Fox River at I	107	1.05	-0.9	0.63	0.66
Fox River at Rochester	289	1.07	-2.5	0.71	0.71
Fox River at Waterford	85	1.06	2.8	0.36	0.73
Fox River at Waukesha	22	1.05	-0.8	0.91	0.92
Fox River near New Munster	216	1.05	-0.2	0.86	0.88
Honey Creek	34	1.05	4.5	0.64	0.86
Mukwonago River at Mukwonago	144	1.04	-0.2	0.88	0.89
Muskego Lake	38	1.11	-10.6	0.54	0.56
Sugar Creek	36	1.06	13.4	0.82	0.98
White River	37	1.08	4.0	0.46	0.74
White River at Lake Geneva	78	1.11	-5.2	0.67	0.71
Wind Lake	36	1.09	-11.6	0.84	0.89

In addition to quantitative performance statistics, visualizations can also aid in diagnosing problems in flux models (Hirsch, 2014). Systemic biases are apparent when model residuals are plotted against estimations and each of the covariates, discharge, time, and season. Biases can also be diagnosed when samples occur in a frequency that does not align with natural variation—boxplots of the variation between sample concentrations and estimates are useful for testing differences in these distributions. Similarly, these biases are apparent if boxplots are created for values of discharge on sampled days versus all daily discharge values. Simple scatterplots showing observed versus simulated for both concentration and flux, can also be useful. Reviewing these plots in a standardized format (Hirsch, 2014) is a quick way to assess an individual site-specific flux model. Diagnostic plots are provided in Appendix C for TSS and Appendix D for TP.

In addition to diagnostic plots, time-series plots showing continuous daily flux estimates along with sampled flux (sample concentration multiplied by daily mean discharge), can reveal specific times when large errors occurred. The time-series plots for all TSS sites are provided in Appendix E, and time-series plots for all TP sites are provided in Appendix F.

4.6. Additional Load Estimations

Data from the Des Plaine River were not included in the linear mixed effects model described in the previous sections. The Des Plaines River has a large area with tile drainage, which impacts the phosphorus dynamics in the watershed. Concentrations and loads for the Des Plaines River were independently estimated using the standard form of LOADEST (Schwarz, Hoos, Alexander, & Smith, 2006) within the rloadest R package (Runkel & De Cicco, 2017). LOADEST model 6 was used for estimating loads and flows for the Des Plaines River. The model equation is provided in Equation 4.5.

$$\ln(P) = a_0 + a_1 \ln(Q) + a_2 \ln(Q^2) + a_3 \sin(2\pi T) + a_4 \cos(2\pi T) \quad \text{Equation 4.5}$$

In the above equation, P represents TP concentration, a_0 through a_3 are model coefficients, Q is flow, and T is decimal time.

Concentration and load estimates at one additional site were also determined for the TMDL. Illinois EPA maintains a chemistry monitoring station on the Fox River just south of the Wisconsin border. The station is identified as DT-35 and contains frequent TP concentration data. Water chemistry data for station DT-35 (IL_EPA_WQX_DT-35) from 2001 to 2022 were downloaded from the Water Quality Portal (National Water Quality Monitoring Council, 2024). Flows at DT-35 were estimated from continuous flow data at the USGS flow gage on the Fox River at New Munster (05545750). The flows from the Fox River at New Munster were scaled using the drainage area method, which is the flow rate multiplied by the ratio of drainage area upstream of DT-35 (859 mi²) and the drainage area upstream of the Fox River at New Munster (796 mi²). A LOADEST model for TP at DT-35 was developed using the measured TP concentrations and the estimated flow rates. LOADEST model 7 was used for estimating loads and flows. The model equation is provided in Equation 4.6.

$$\ln(P) = a_0 + a_1 \ln(Q) + a_2 \sin(2\pi T) + a_3 \cos(2\pi T) + a_4 T \quad \text{Equation 4.6}$$

In the above equation, P represents TP concentration, a₀ through a₃ are model coefficients, Q is flow, and T is decimal time.

Model coefficients for the Des Plaines River and the Fox River at DT-35 are provided in Table 4.5. Fit statistics for two locations are provided in Table 4.6.

TABLE 4.5
Coefficients for TP LOADEST Models

Monitoring Location	Intercept	ln(Q)	ln(Q ²)	cos(2πT)	sin(2πT)	T
Des Plaines River at Russell, IL	-2.29	0.0246	0.0096	-0.580	-0.323	-
Fox River at DT-35 (IL)	-2.52	0.257	-	-0.551	-0.219	-0.013

TABLE 4.6
Fit Statistics for TP LOADEST Models

Monitoring Location	N	BCF	PBIAS	NSE	R ²
Des Plaines River at Russell, IL	38	1.10	-5.8	0.12	0.4
Fox River at DT-35 (IL)	162	1.08	-2.0	0.48	0.63

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APPENDIX A

RATING CURVES AND CONTINUOUS FLOW ESTIMATES

FIGURE A.1
Rating Curve for Fox River at ES

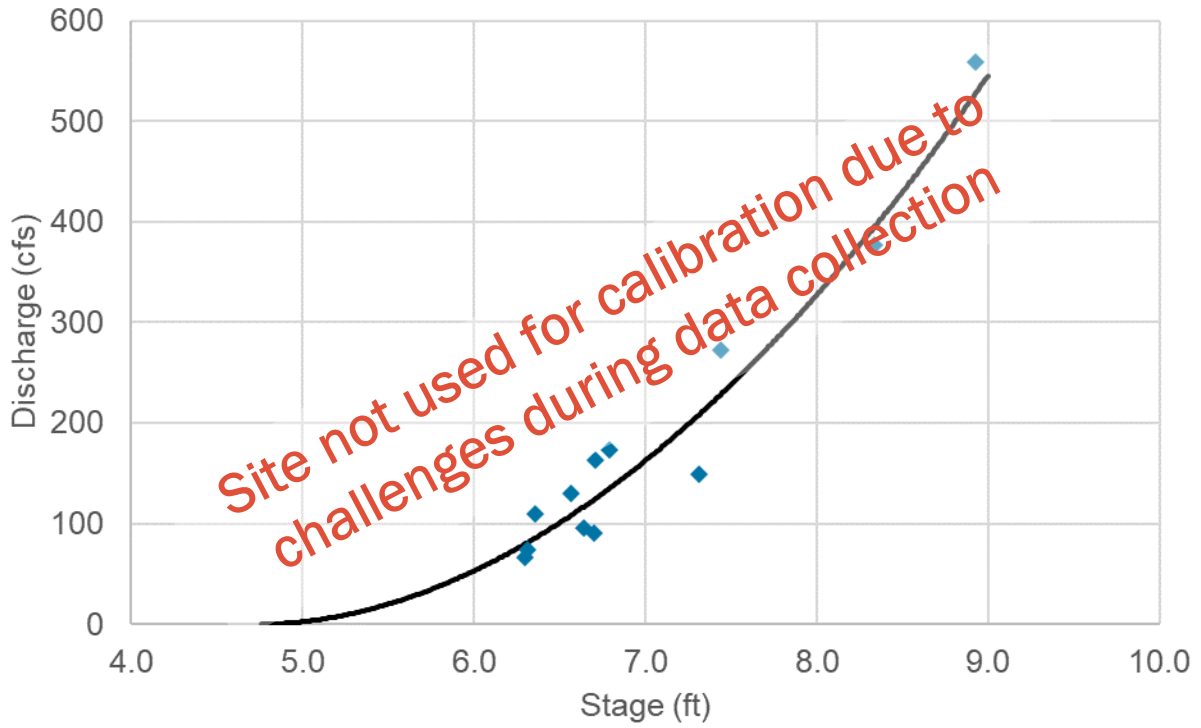


FIGURE A.2
Rating Curve for Honey Creek at Academy Road

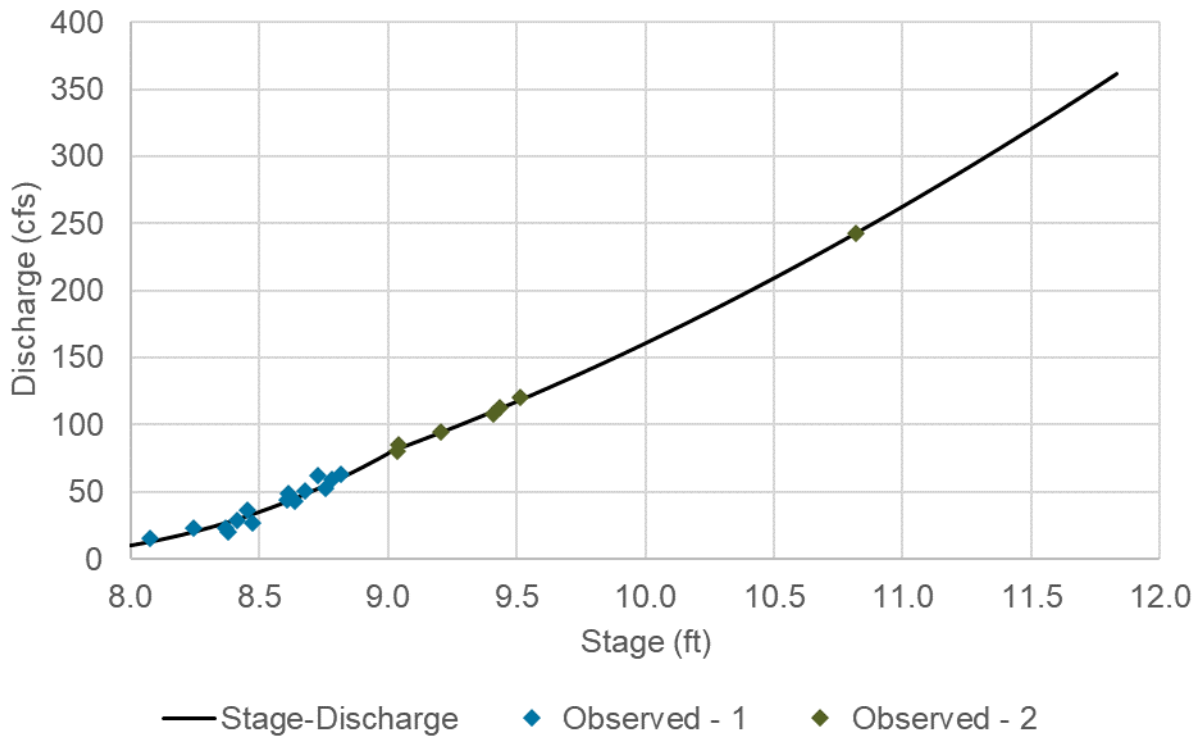


FIGURE A.3
Rating Curve for Sugar Creek at Potter Road

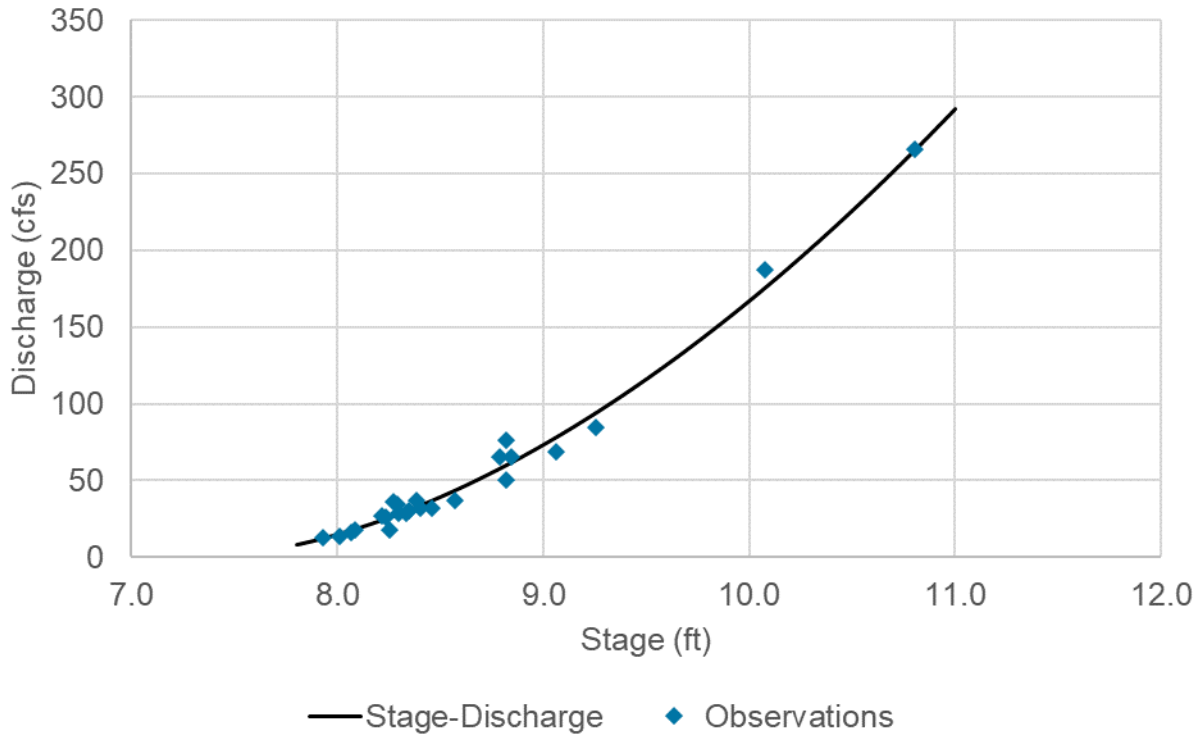


FIGURE A.4
Rating Curve for White River at Hwy 36

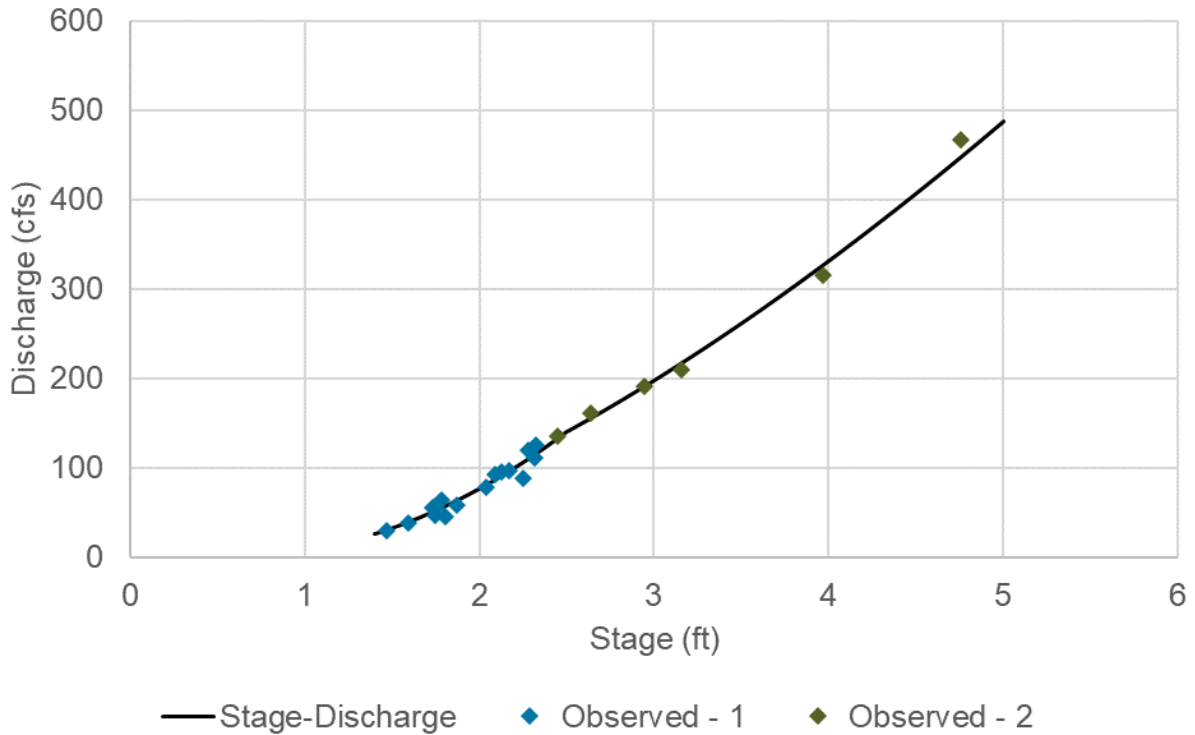


FIGURE A.5

Rating Curve for Muskego Canal at Muskego Dam Road

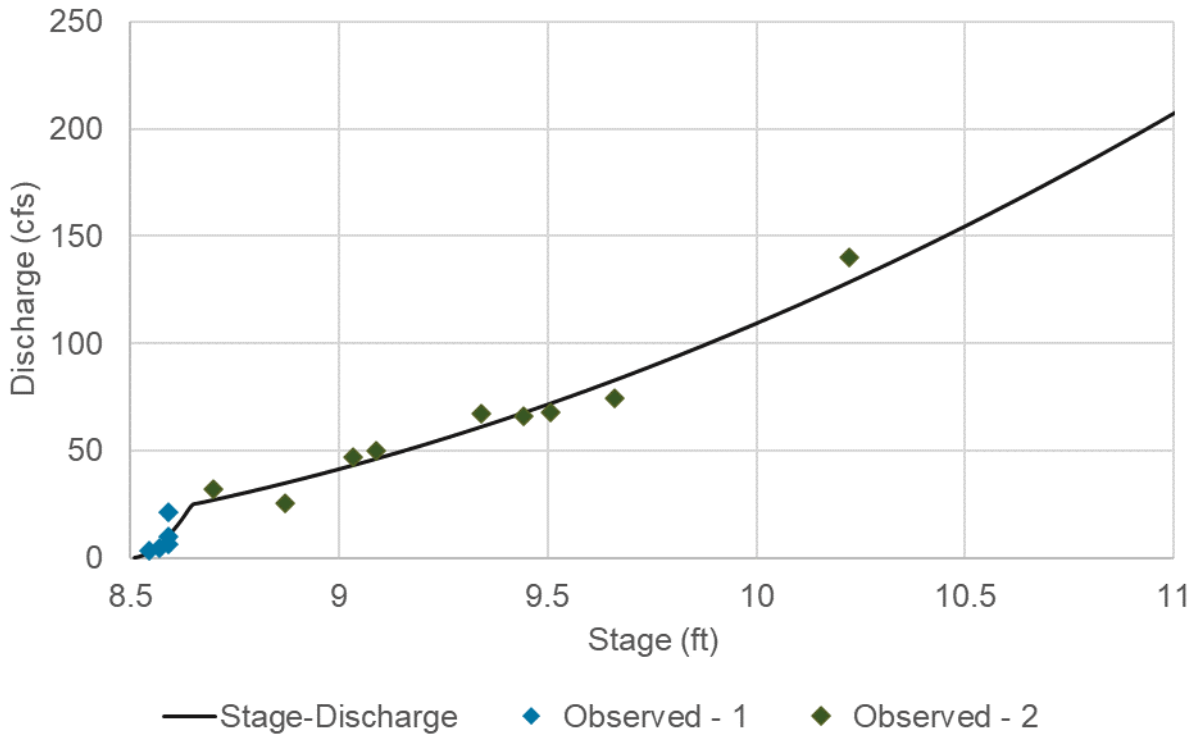


FIGURE A.6

Rating Curve for Wind Lake Outlet at South Wind Lake Road

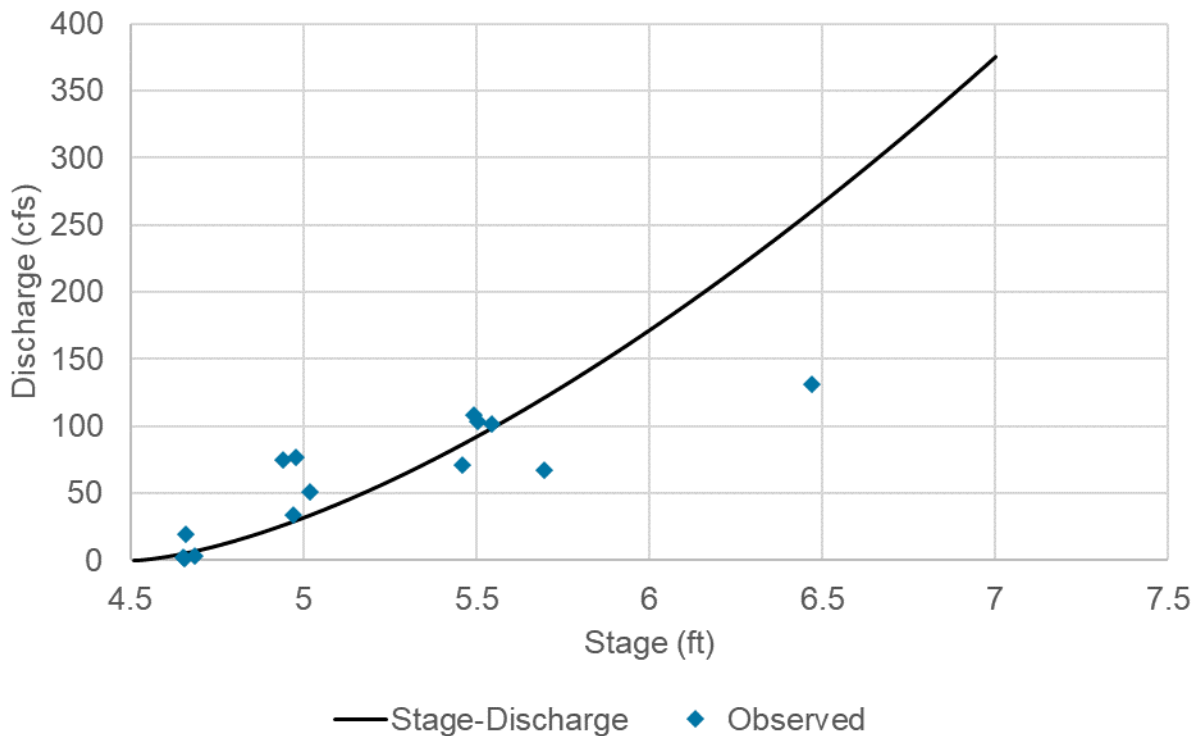


FIGURE A.7
Flow Estimates for Fox River at ES

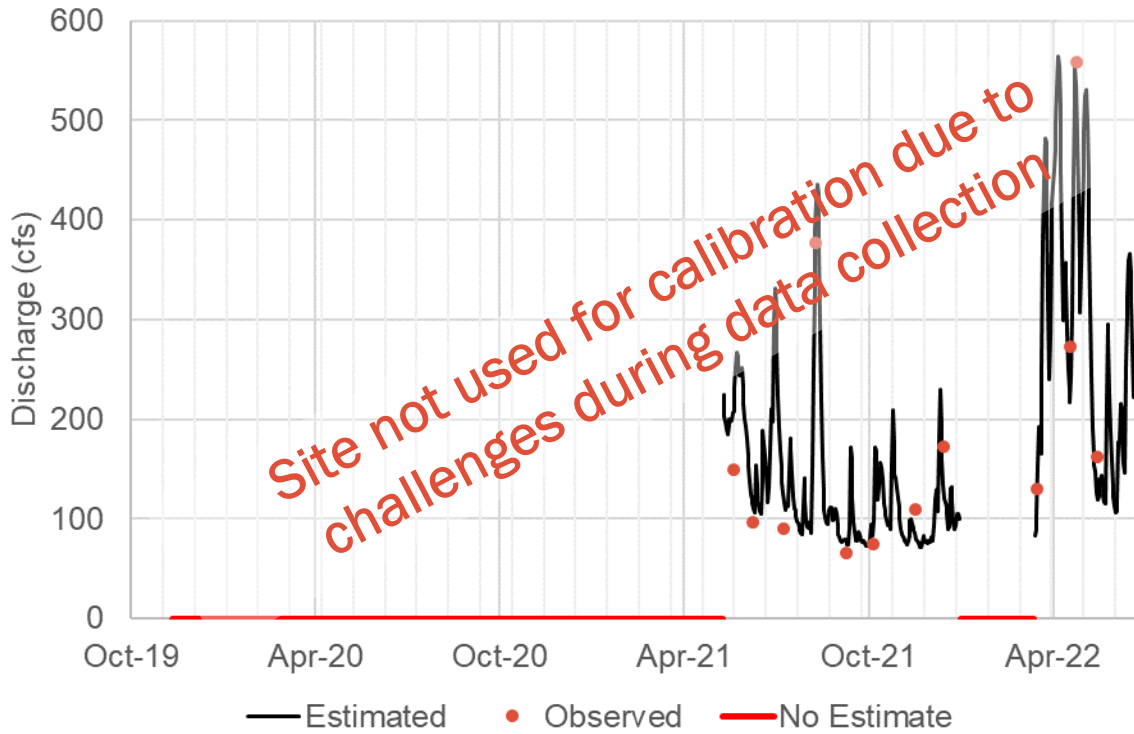


FIGURE A.8
Flow Estimates for Honey Creek at Academy Road

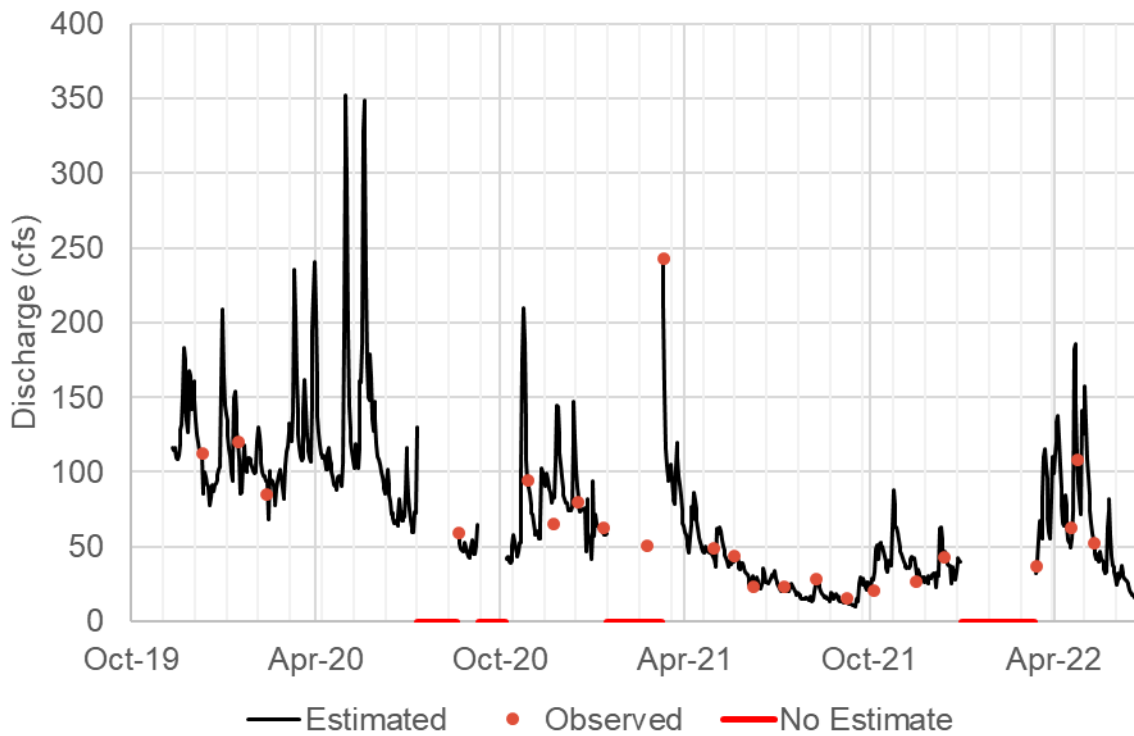


FIGURE A.9
Flow Estimates for Sugar Creek at Potter Road

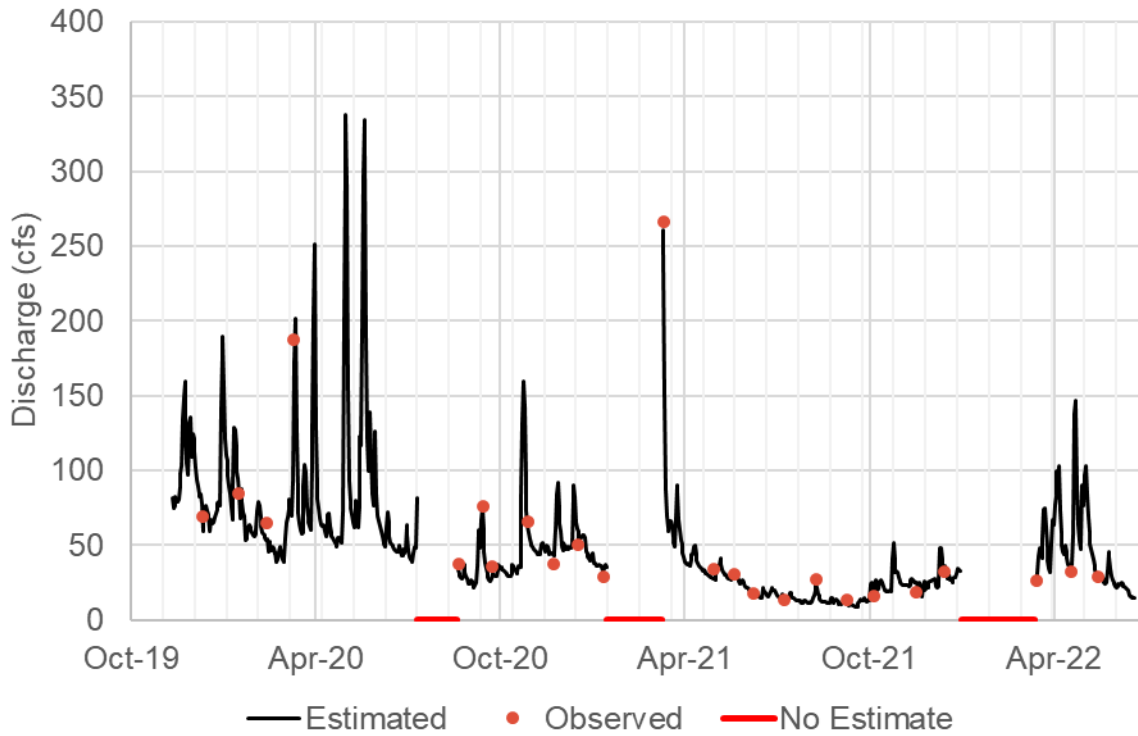


FIGURE A.10
Flow Estimates for White River at Hwy 36

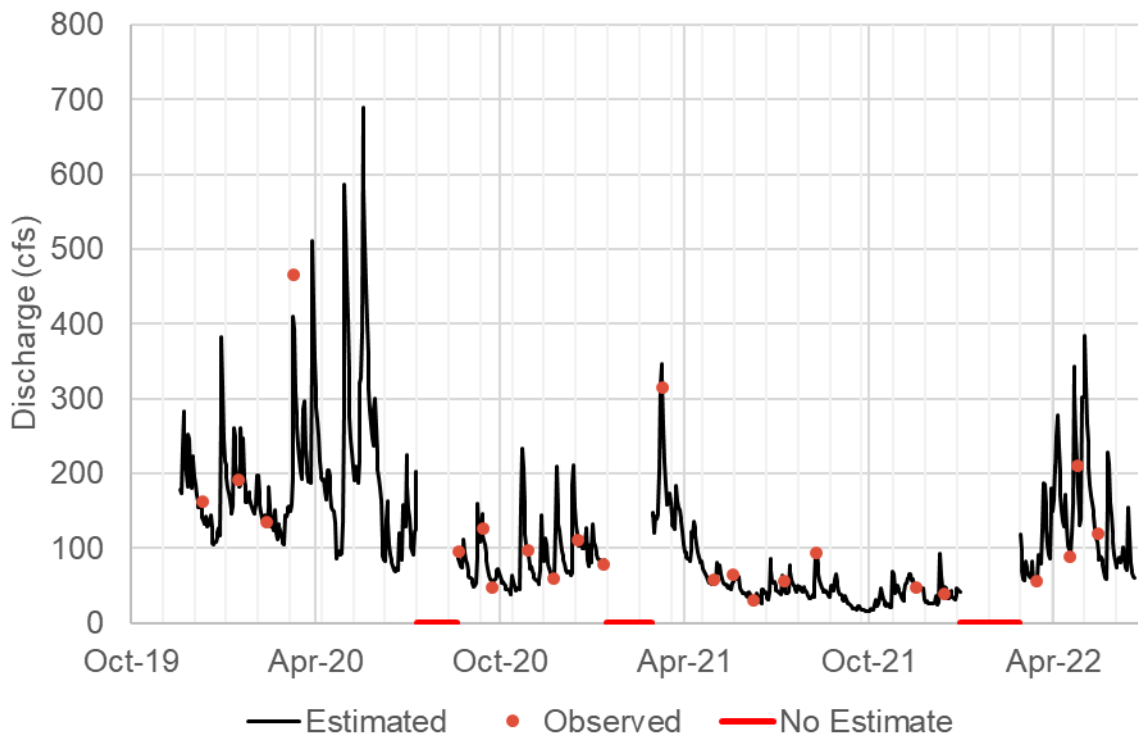


FIGURE A.11

Flow Estimates for Muskego Canal at Muskego Dam Road

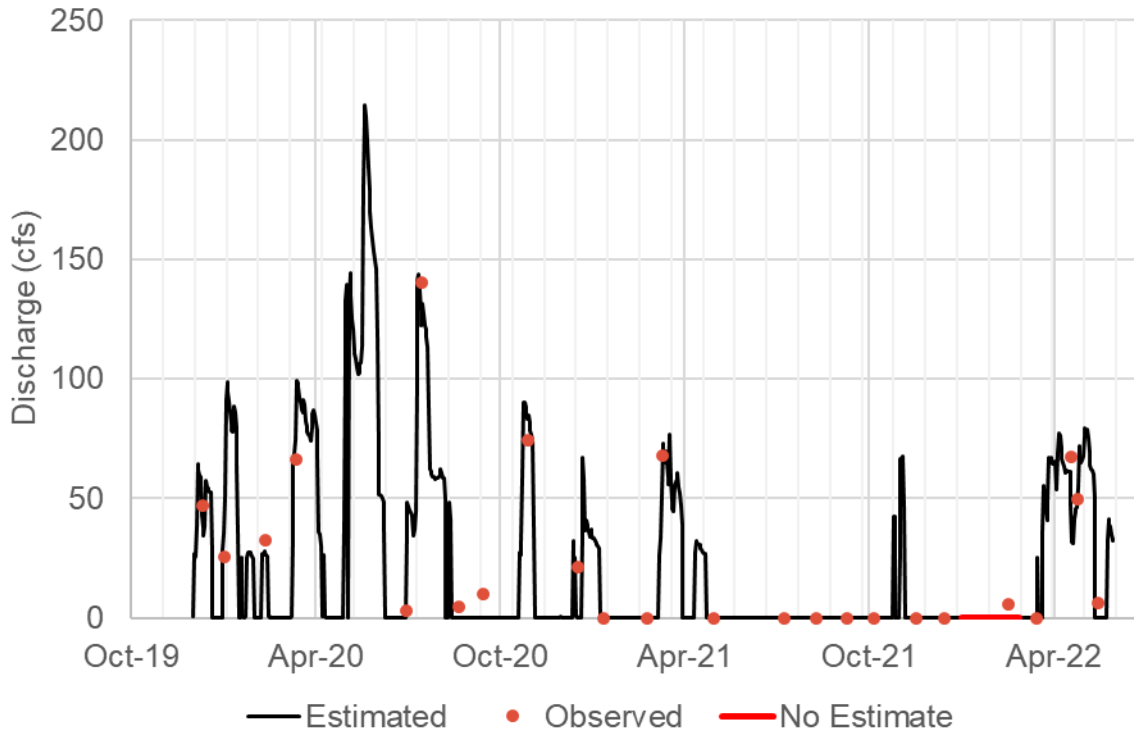
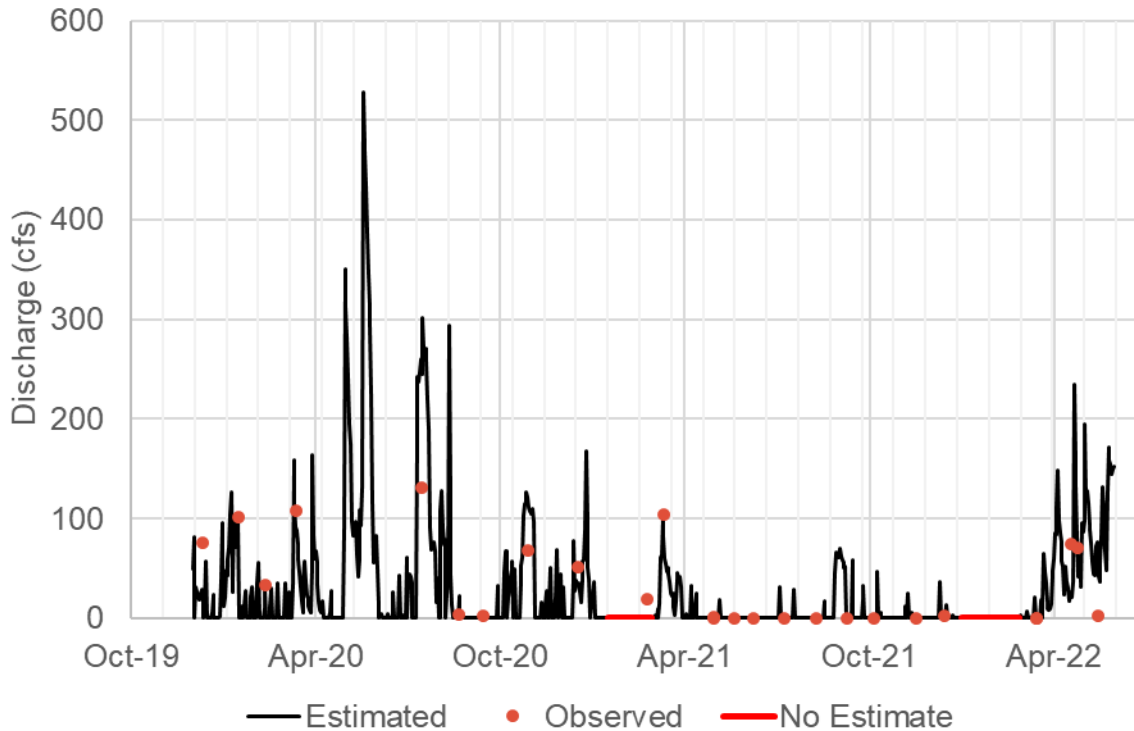


FIGURE A.12

Flow Estimates for Wind Lake Outlet at South Wind Lake Road



APPENDIX B

LINEAR REGRESSION RELATIONS AND CONTINUOUS FLOW ESTIMATES

FIGURE B.1

Relationship between Fox River at Cth I and USGS 05543830

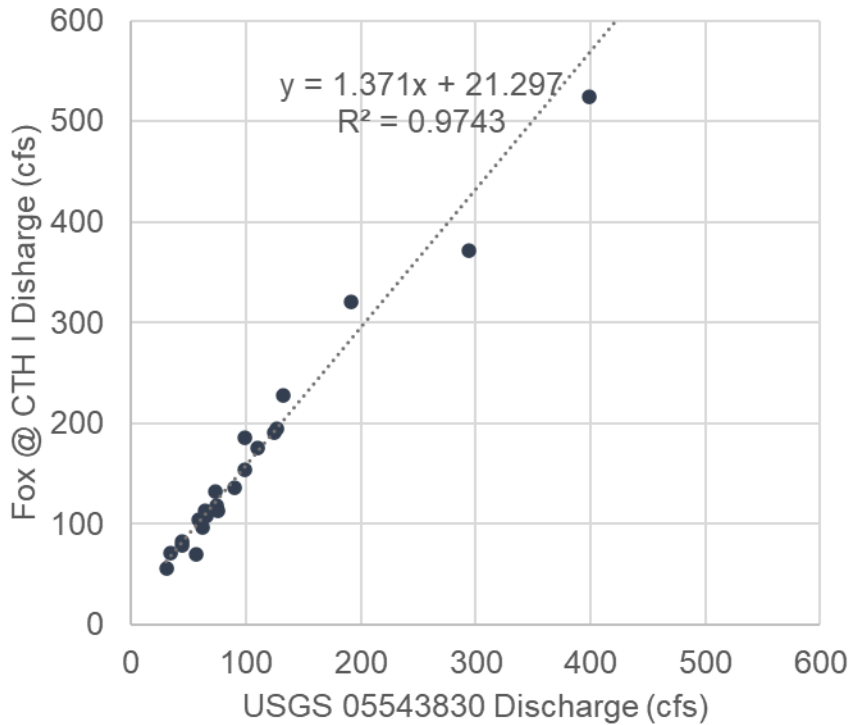


FIGURE B.2

Relationship between Fox River at Waterford and USGS 05545750

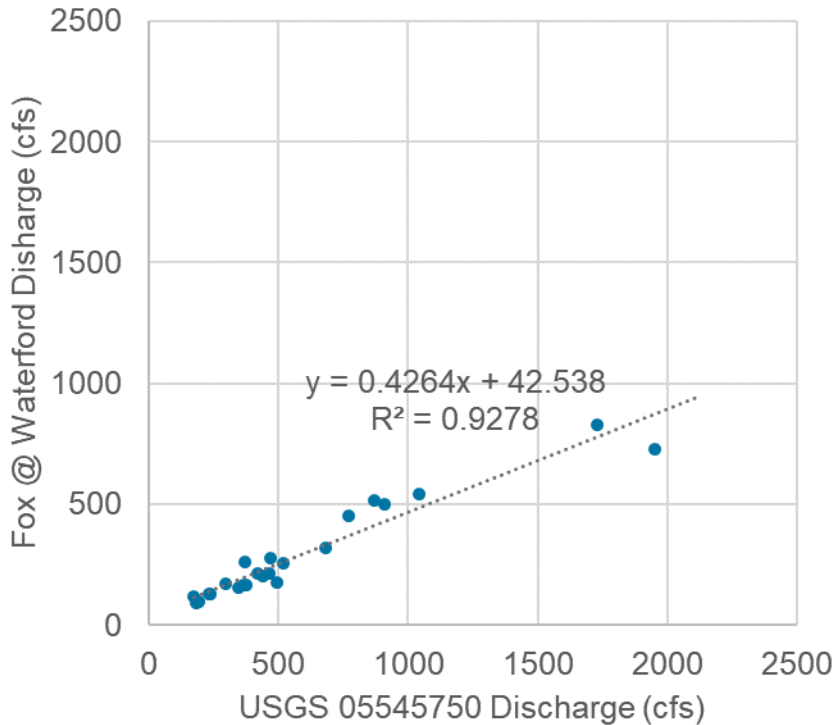


FIGURE B.3

Relationship between Fox River at Rochester and USGS 05545750

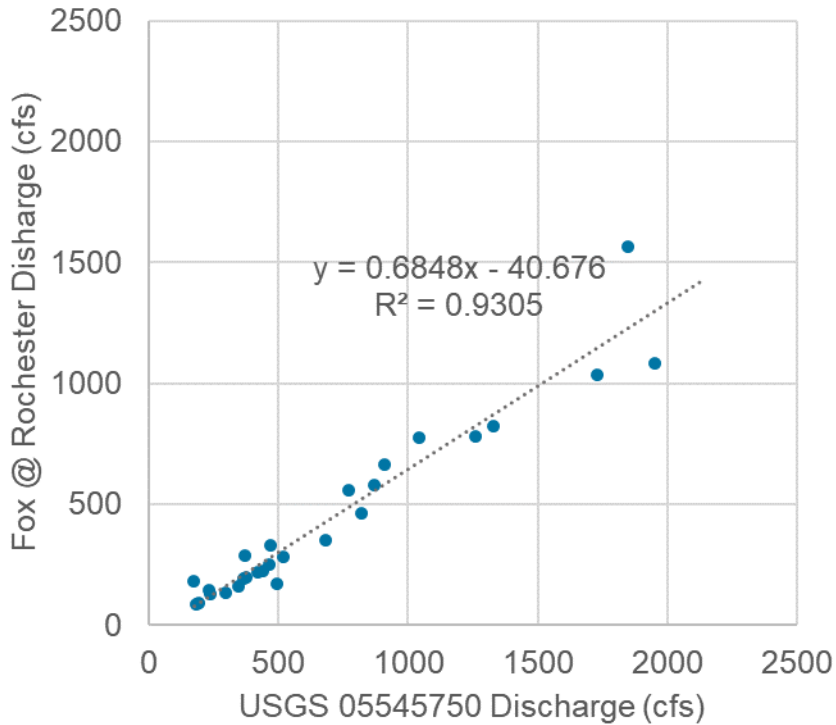


FIGURE B.4

Continuous Flow Estimates for Fox River at Cth I

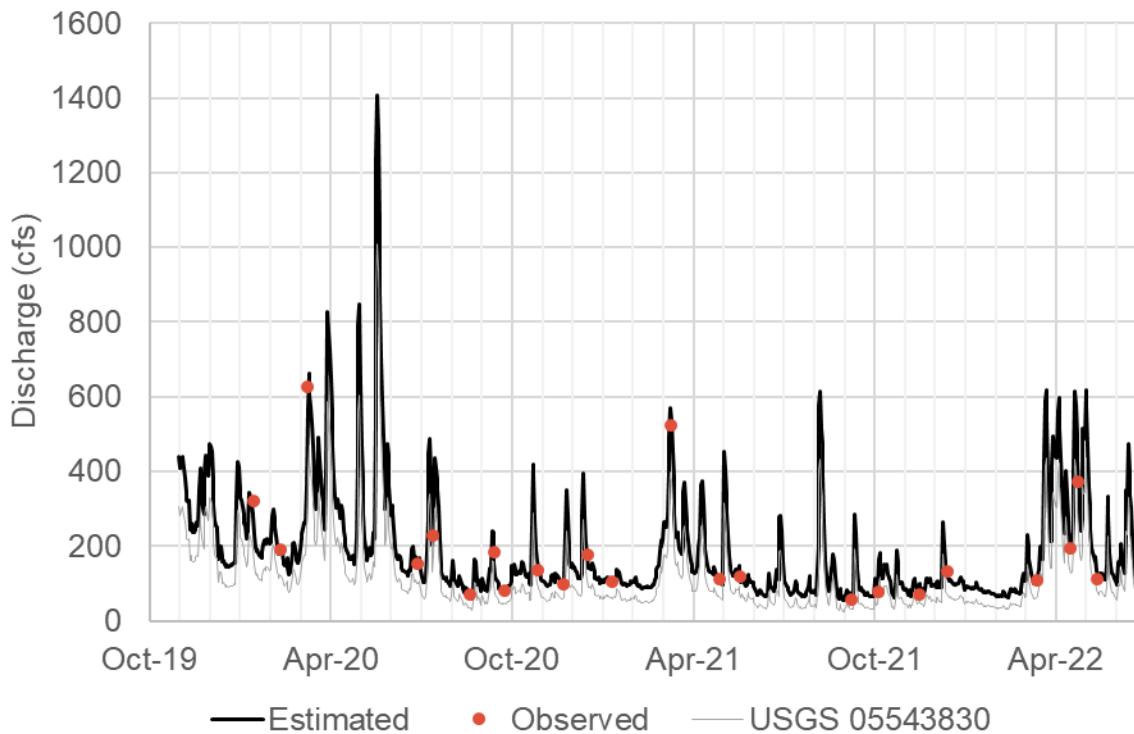


FIGURE B.5

Continuous Flow Estimates for Fox River at Rochester

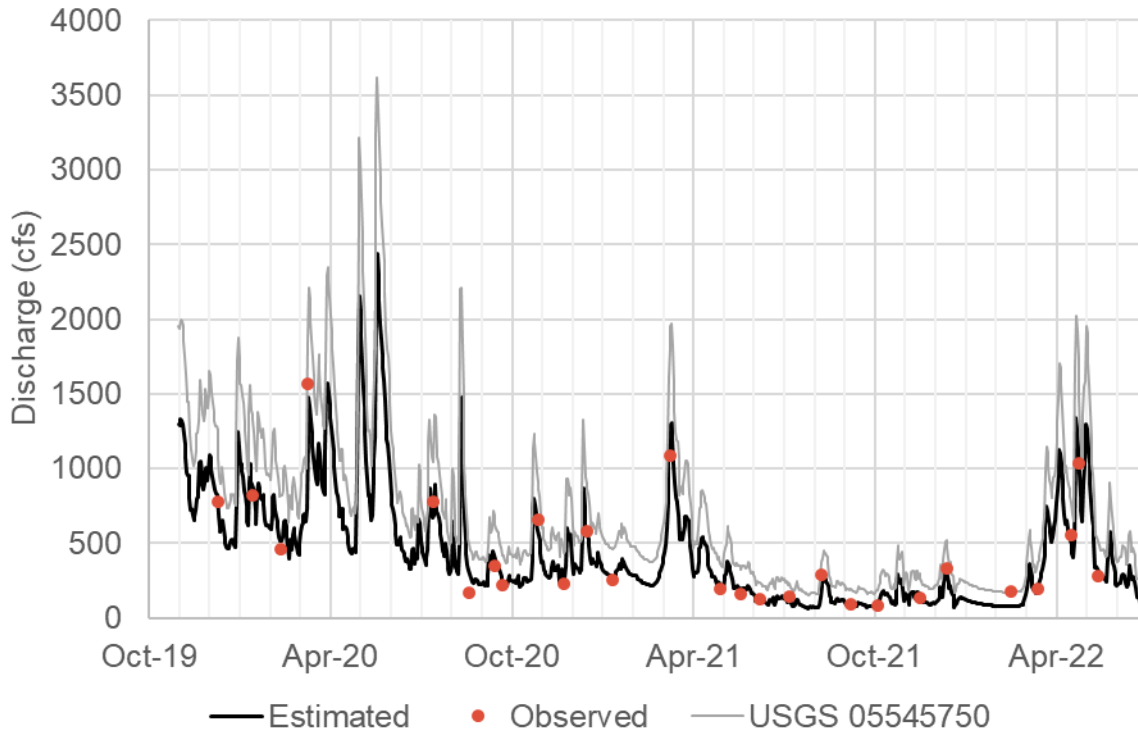
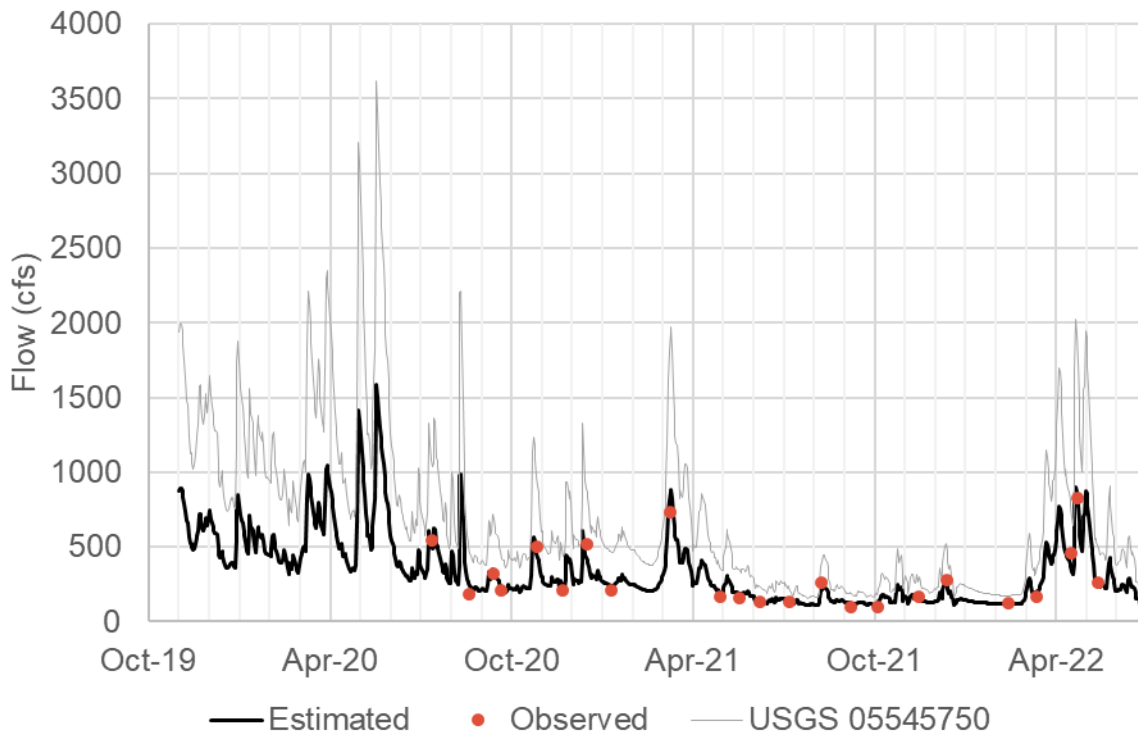


FIGURE B.6

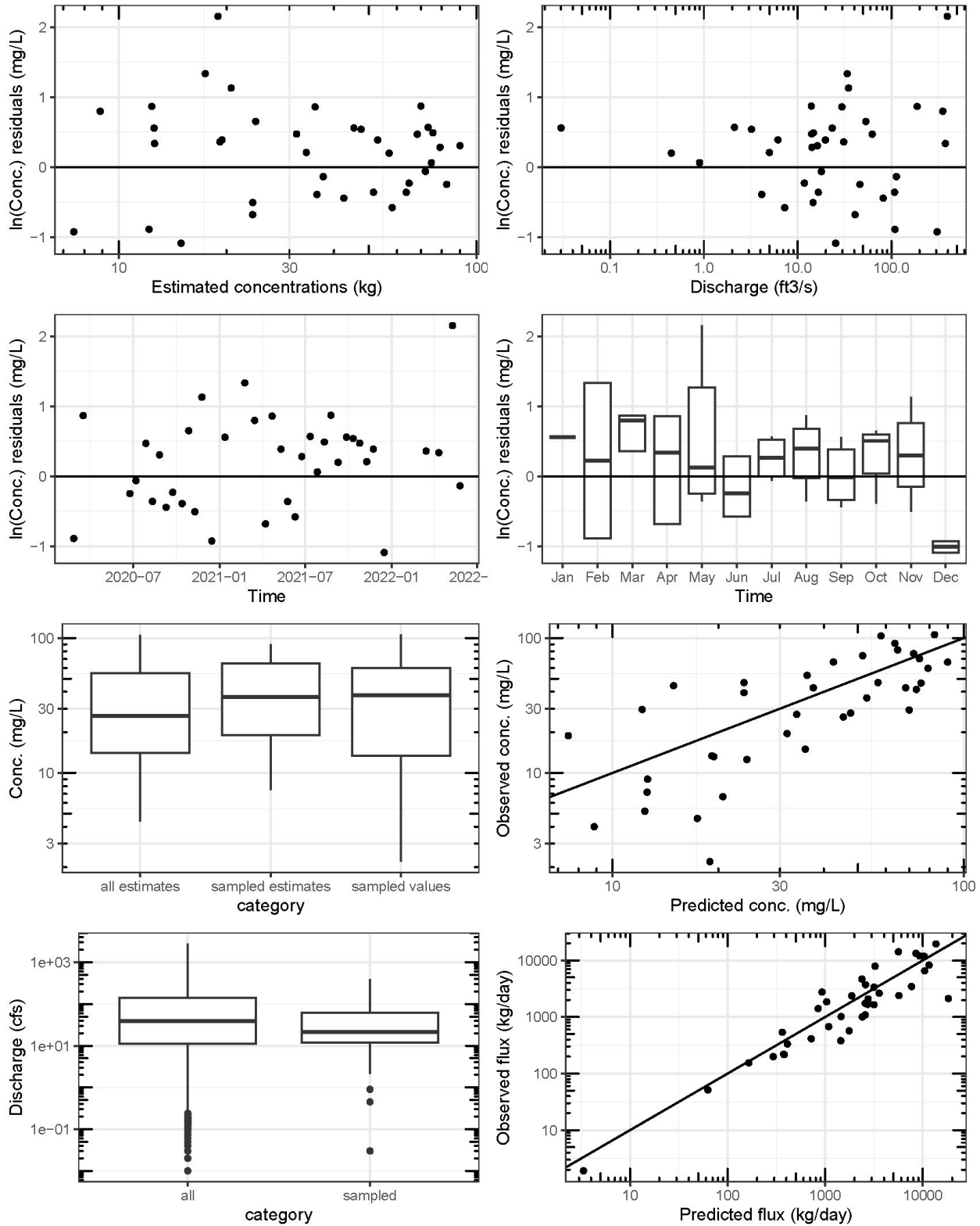
Continuous Flow Estimates for Fox River at Waterford



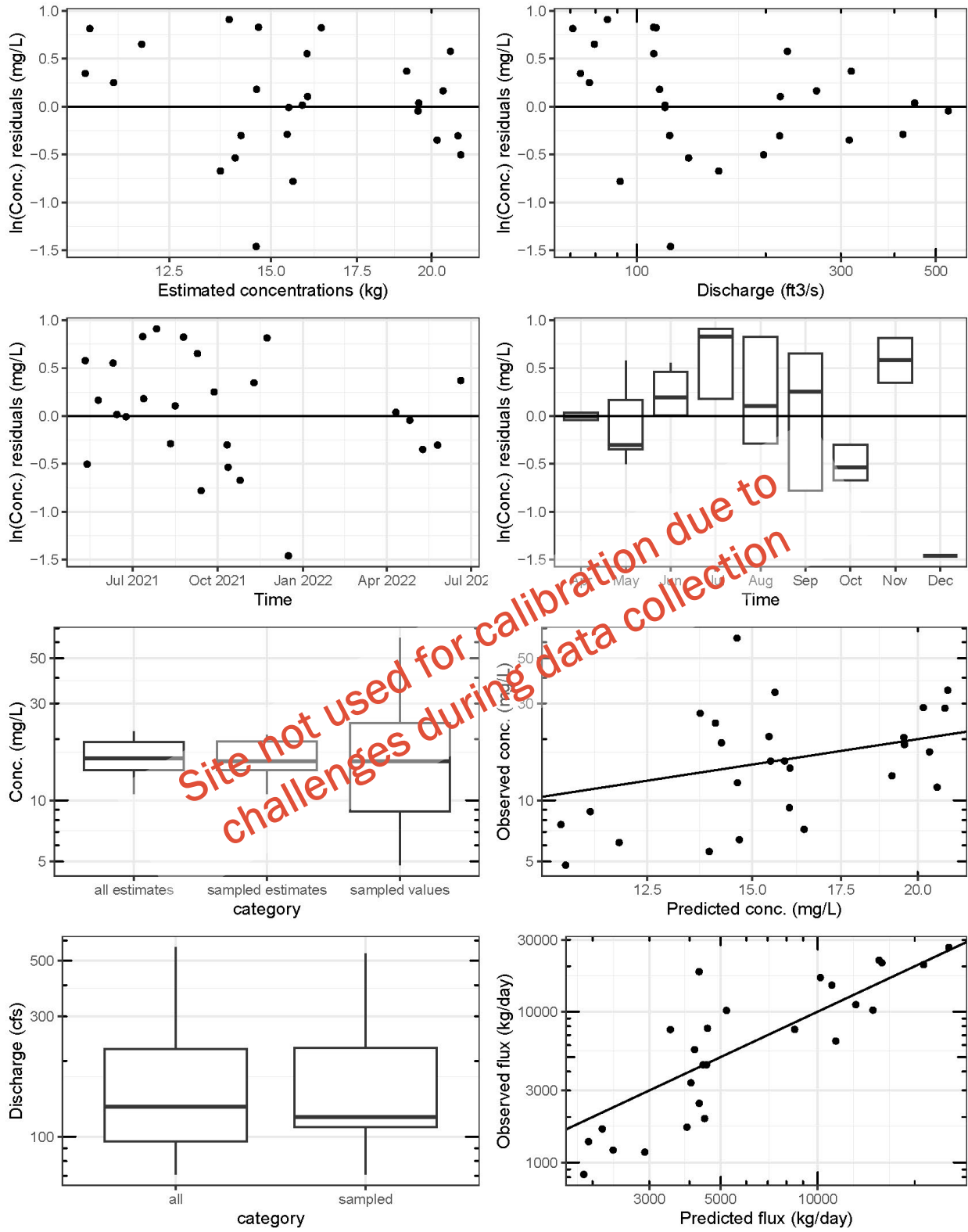
APPENDIX C

DIAGNOSTIC PLOTS FOR TSS LOAD ESTIMATION

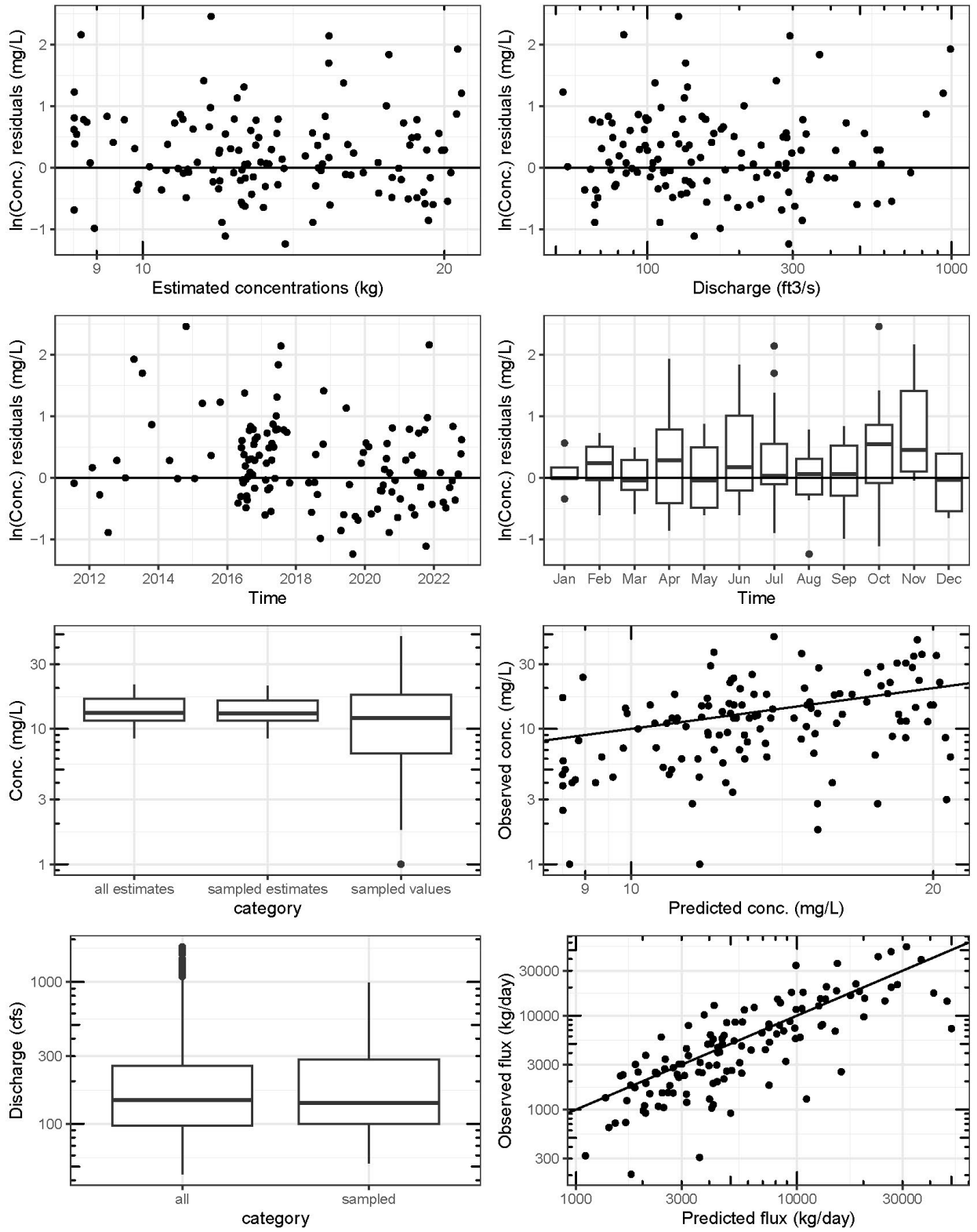
TSS, Des Plaines River at Russell, IL



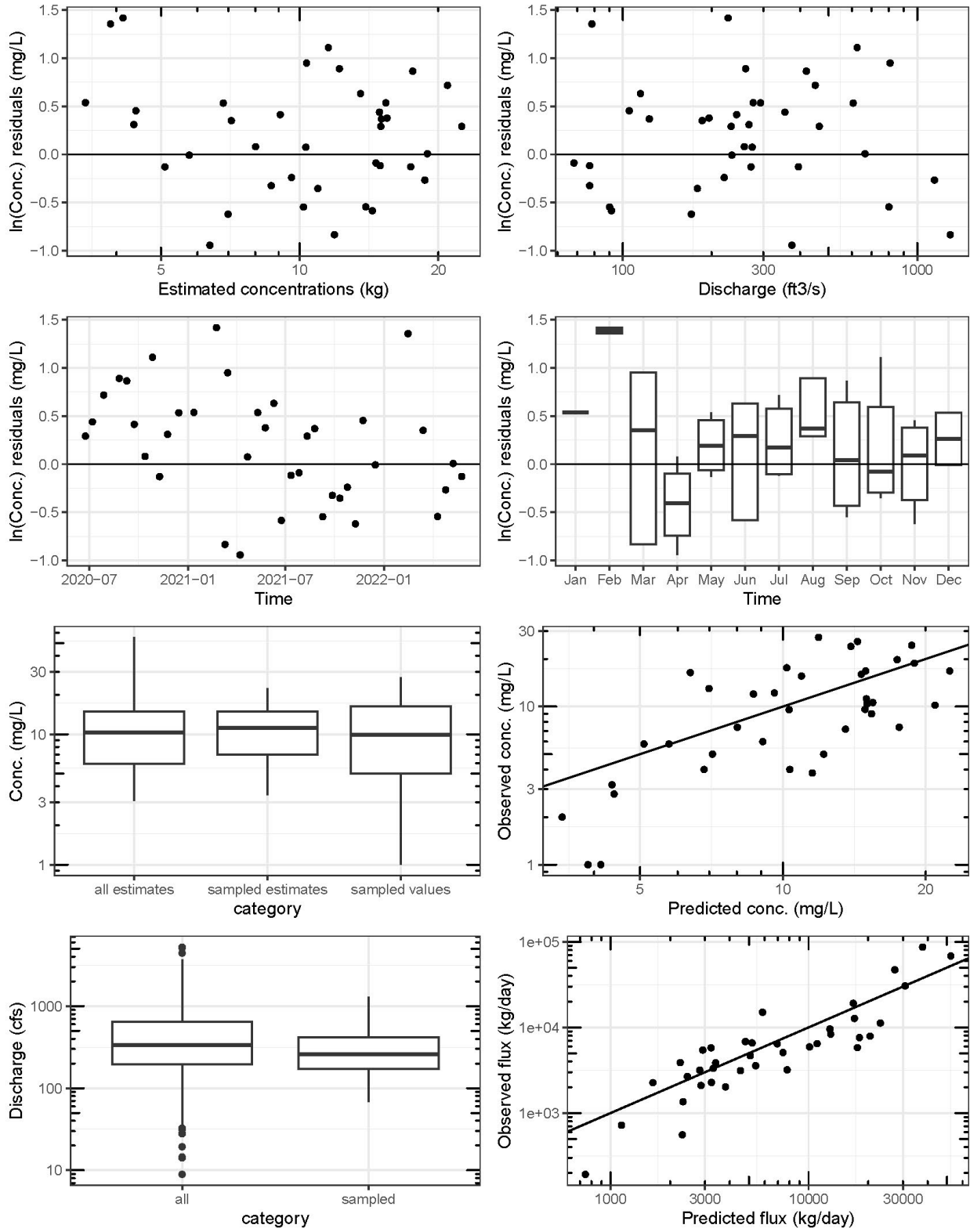
TSS, Fox River at ES



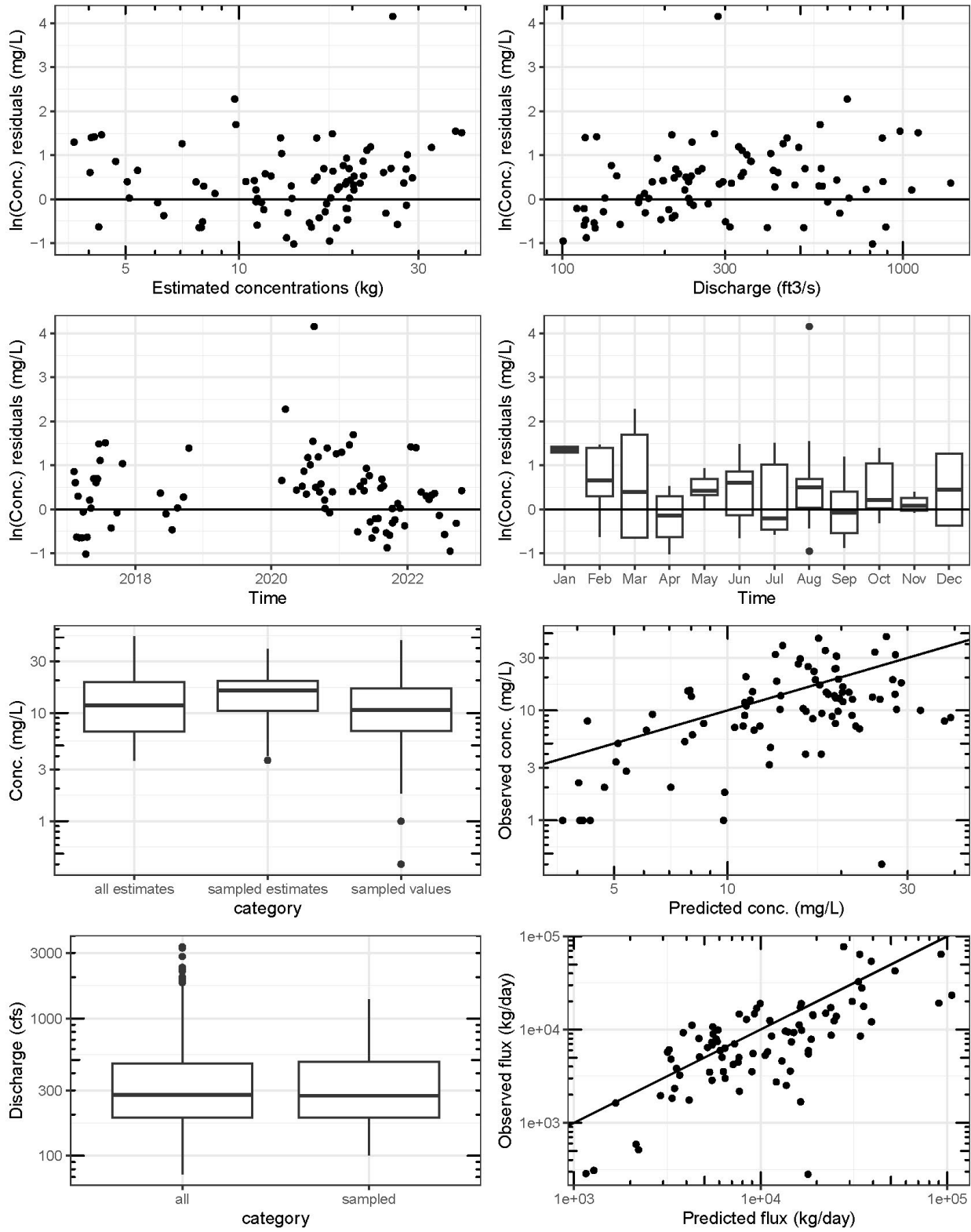
TSS, Fox River at I



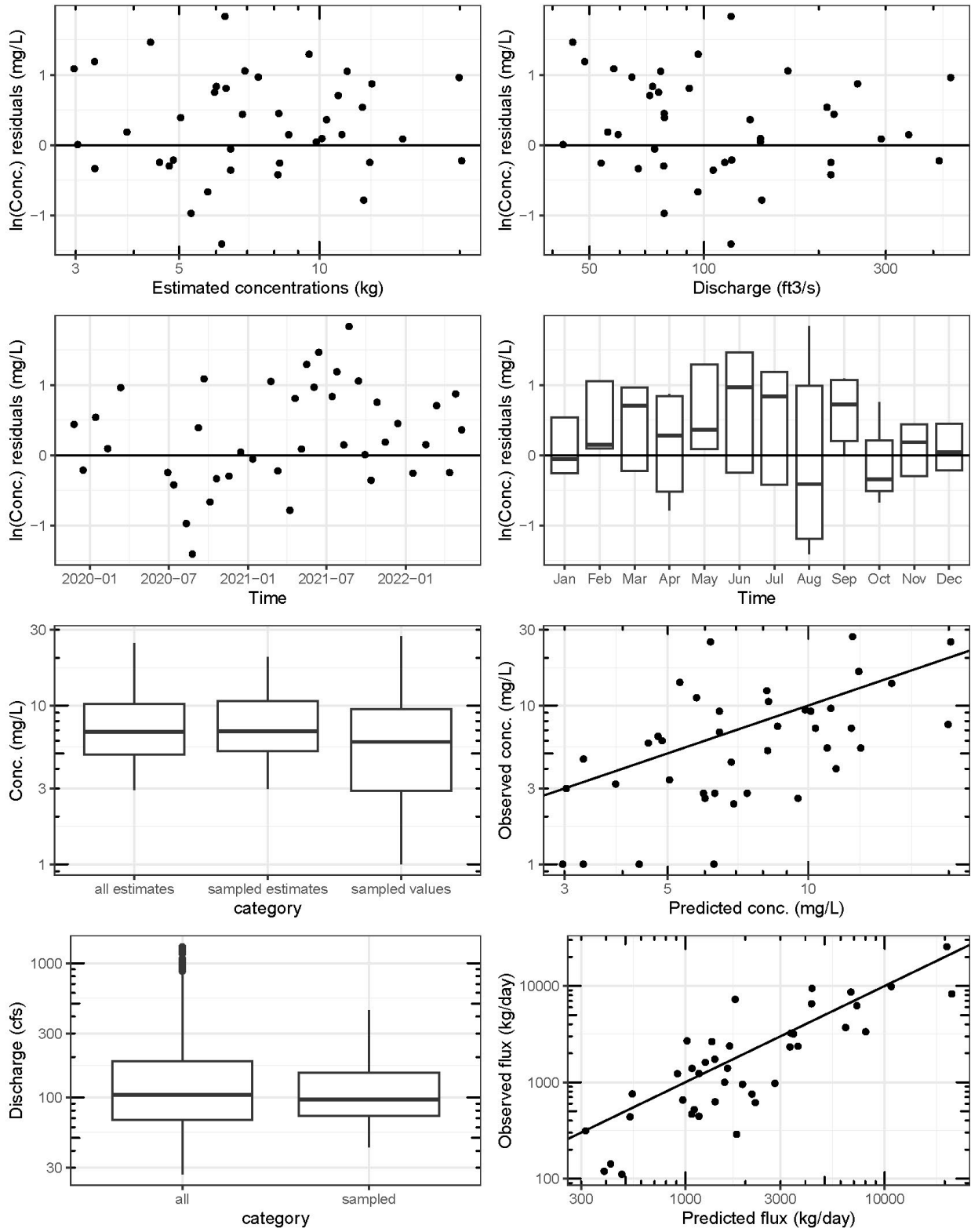
TSS, Fox River at Rochester



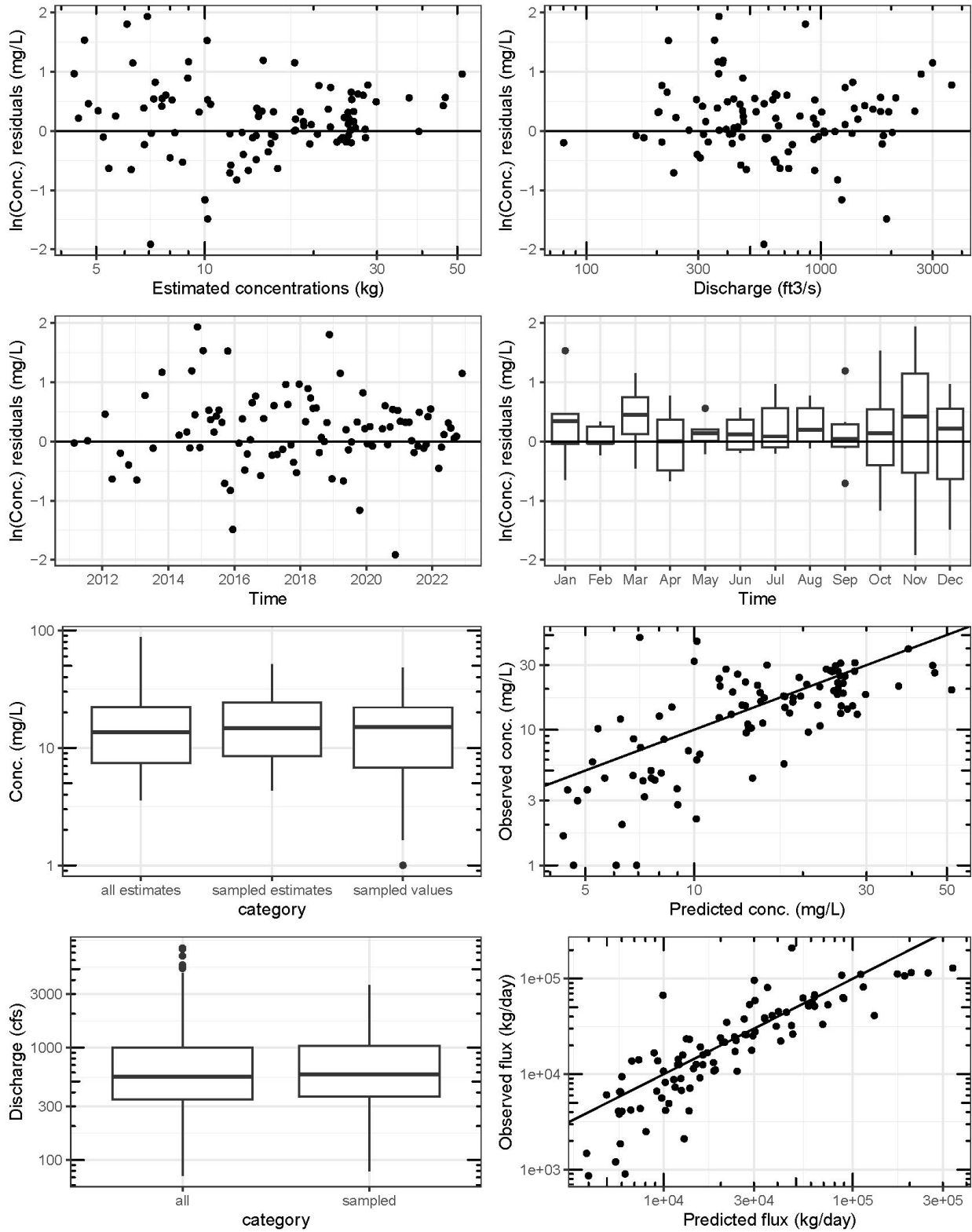
TSS, Fox River at Waterford



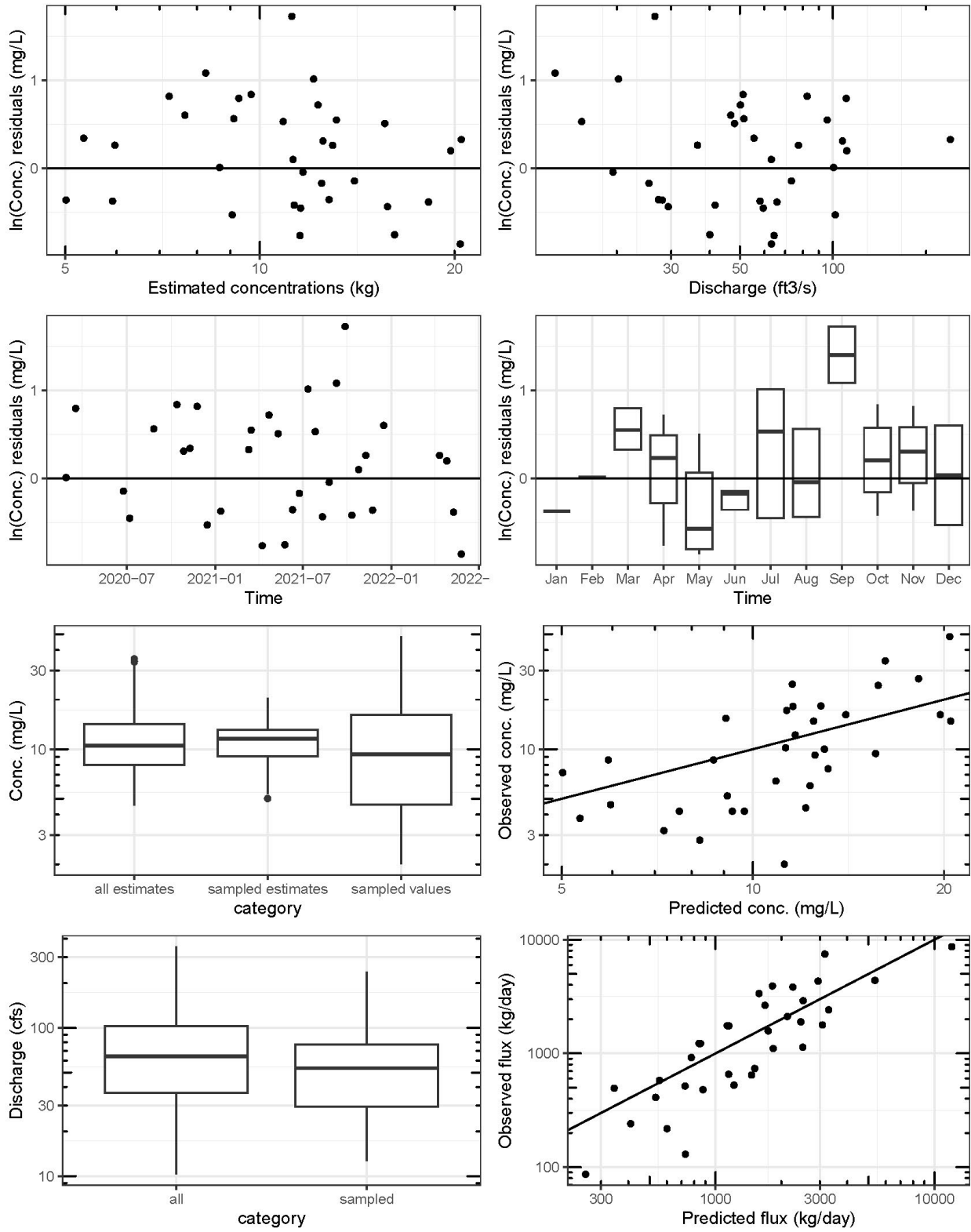
TSS, Fox River at Waukesha



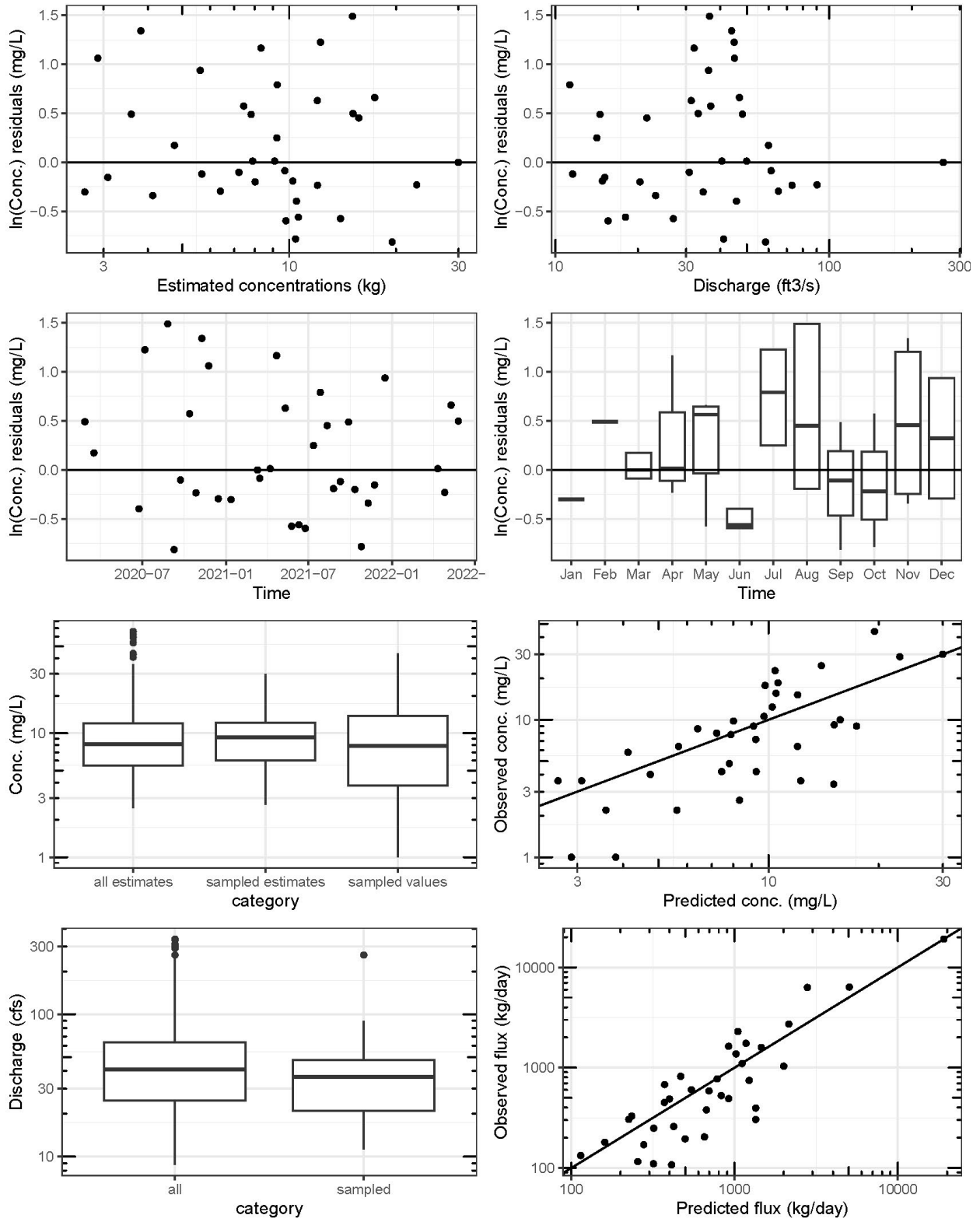
TSS, Fox River near New Munster



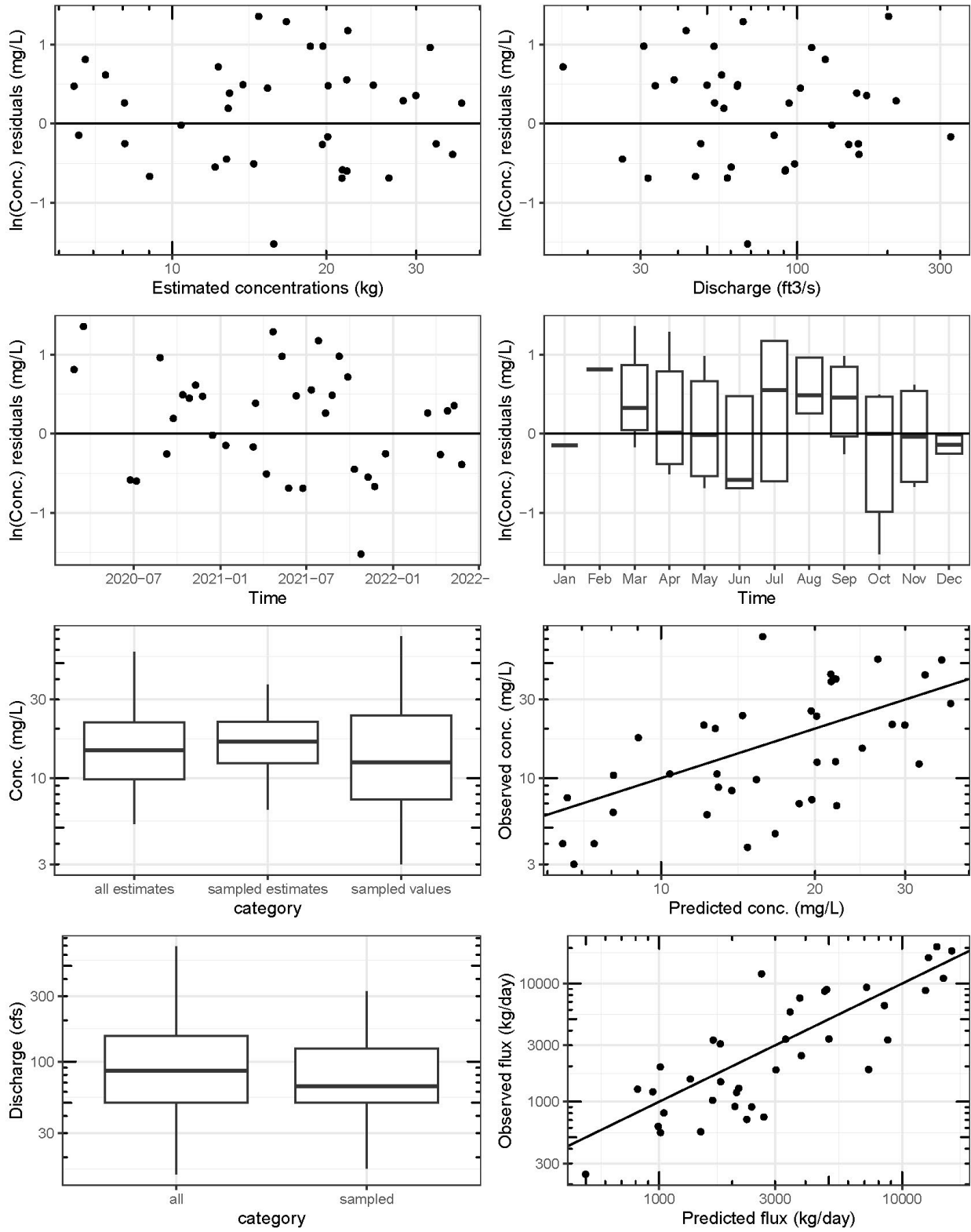
TSS, Honey Creek



TSS, Sugar Creek



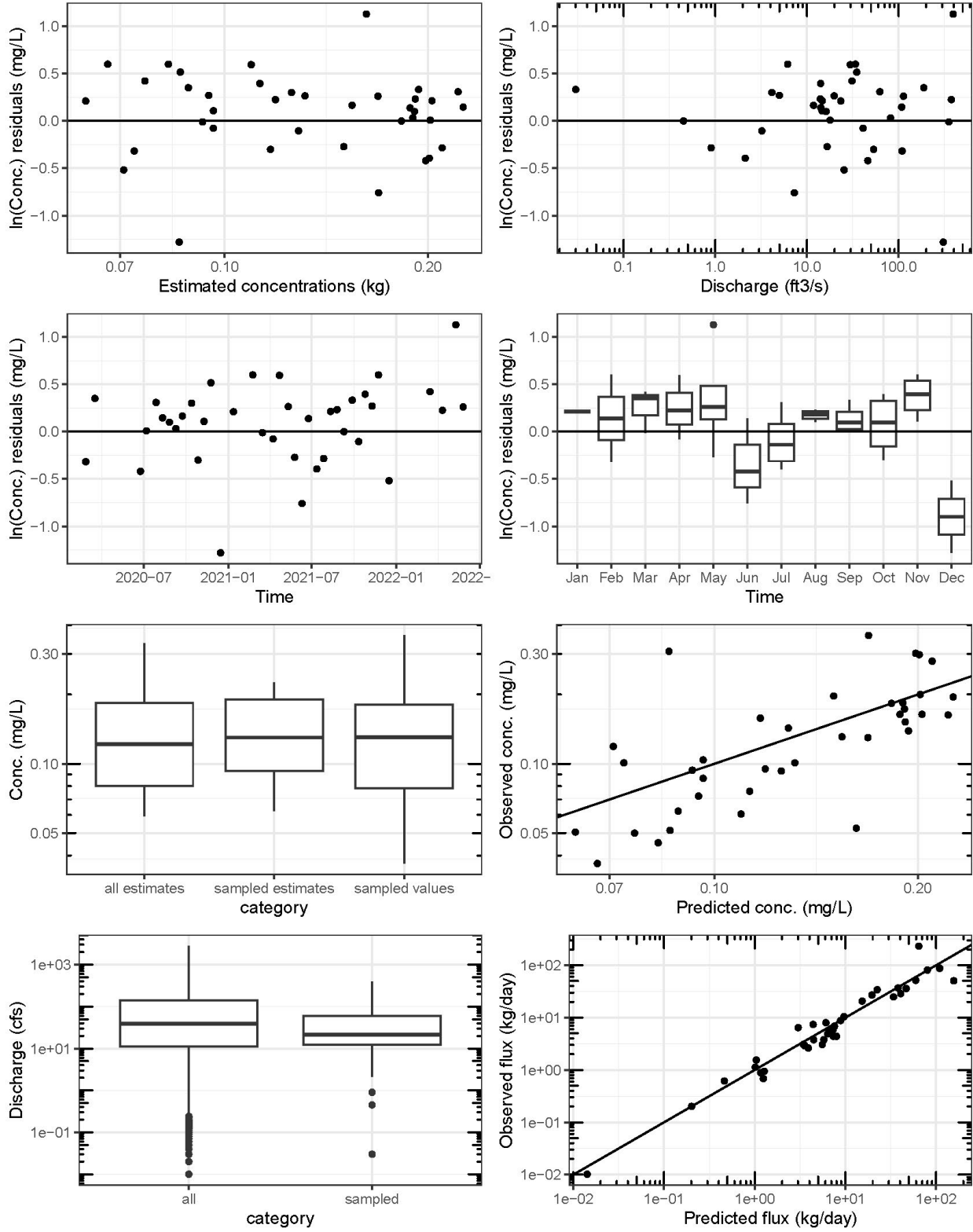
TSS, White River



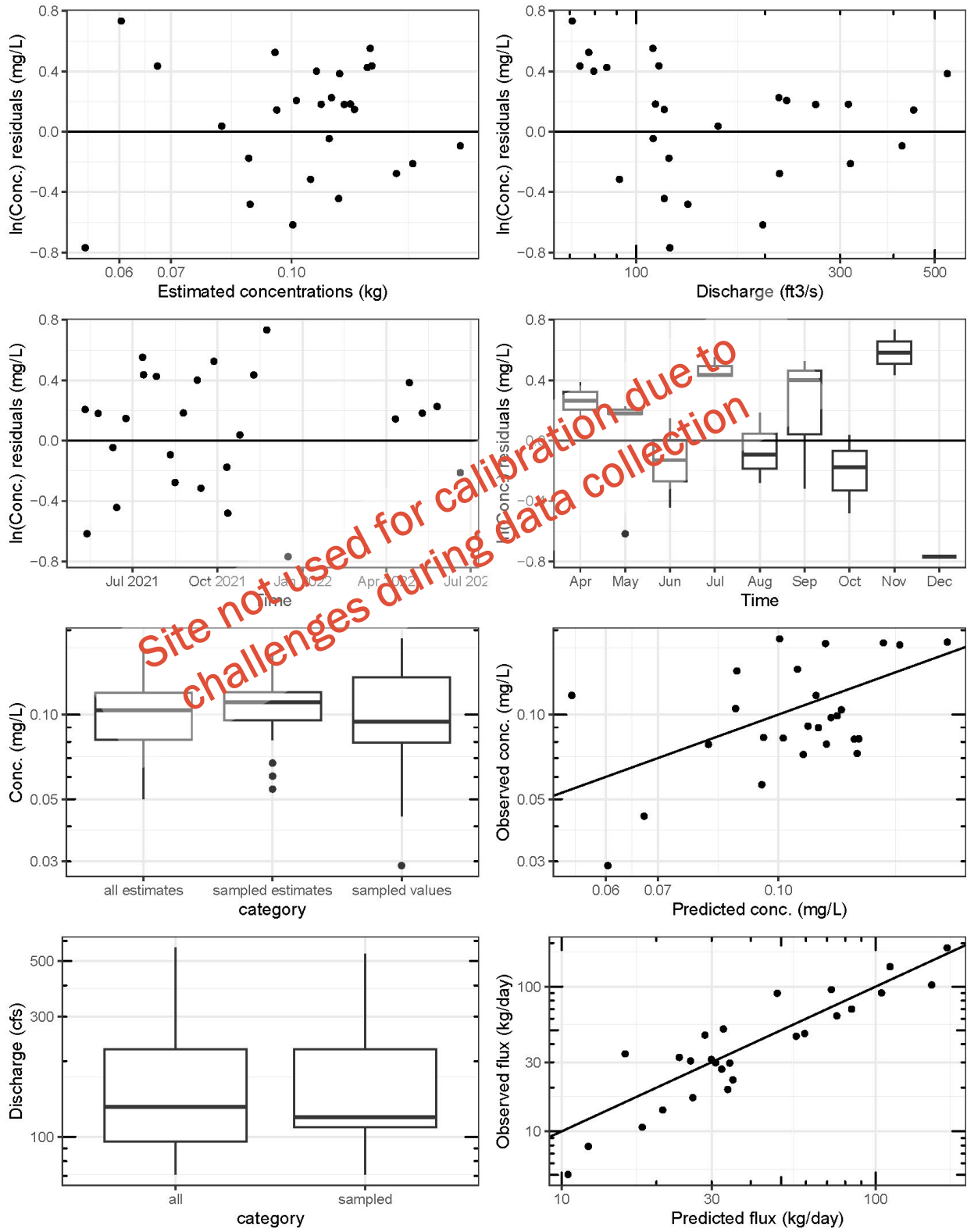
APPENDIX D

DIAGNOSTIC PLOTS FOR TP LOAD ESTIMATION

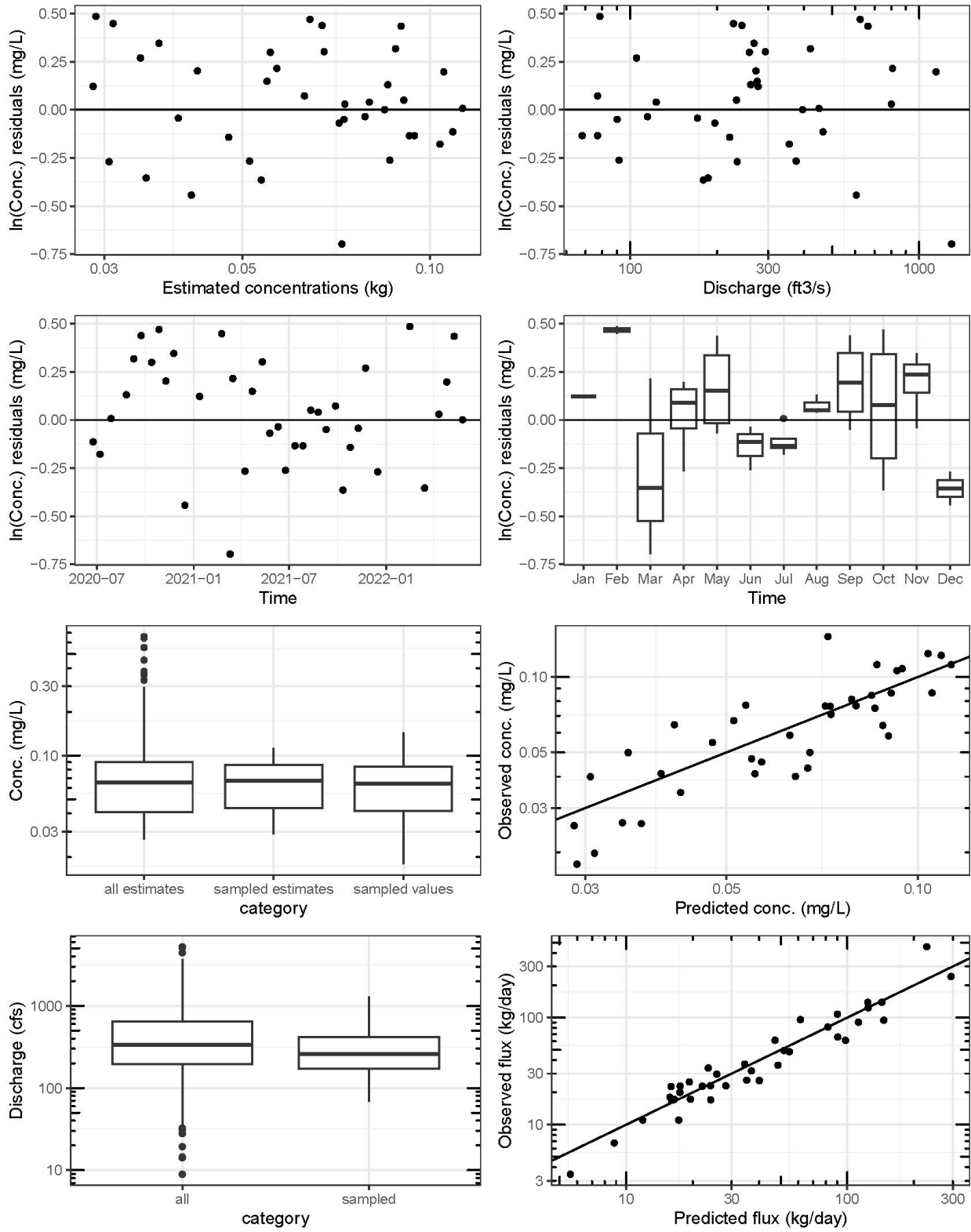
TP, Des Plaines River at Russell, IL



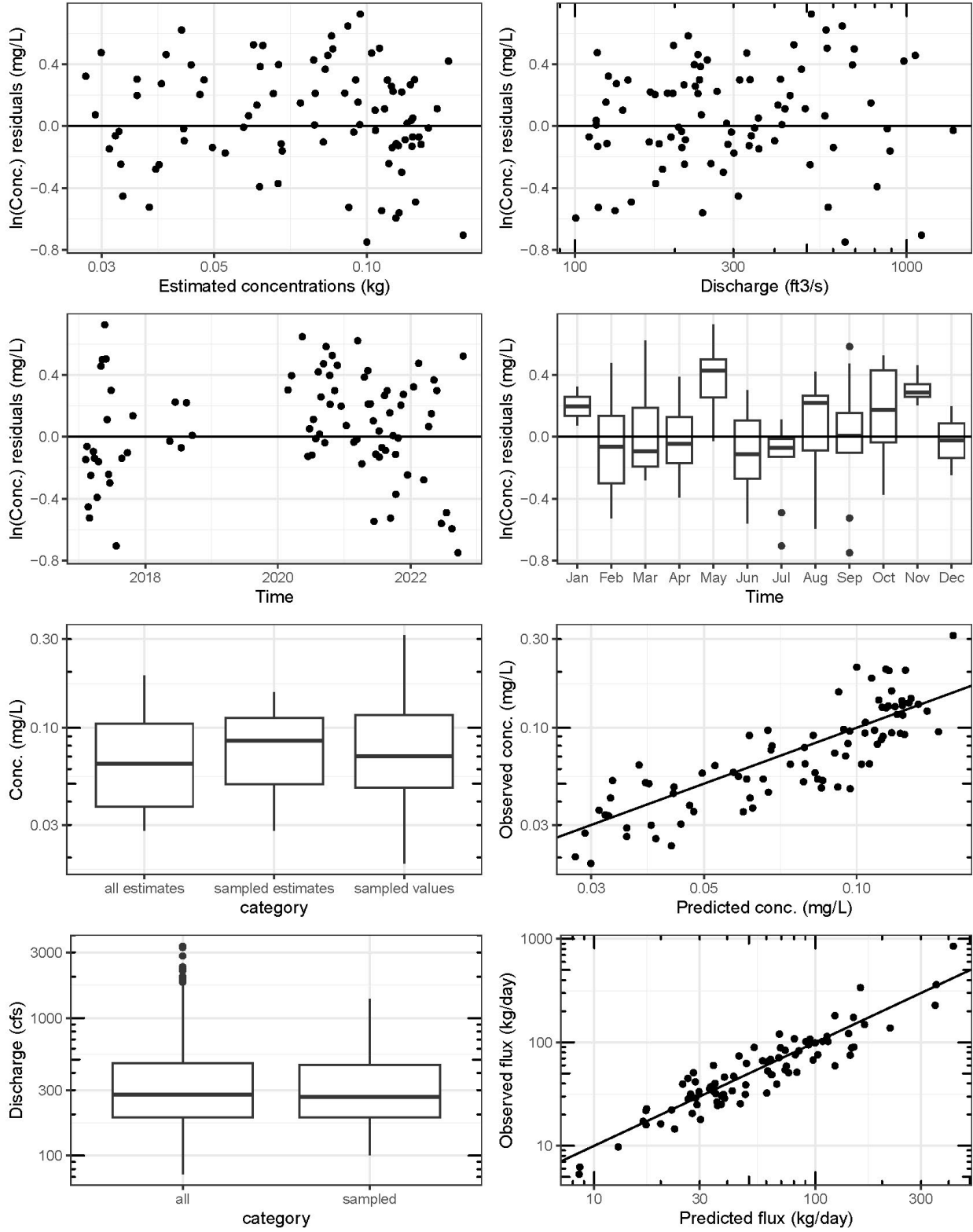
TP, Fox River at ES



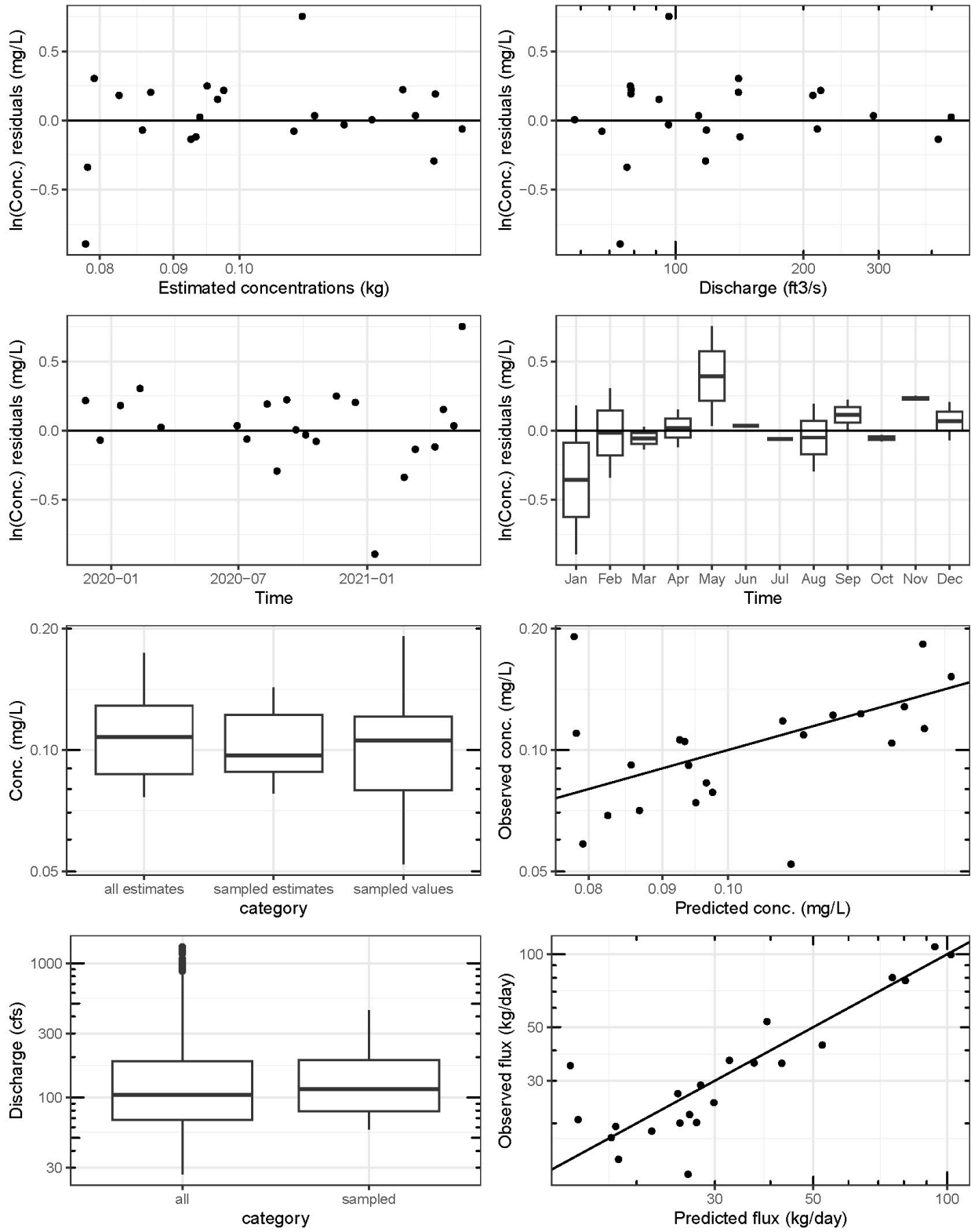
TP, Fox River at Rochester



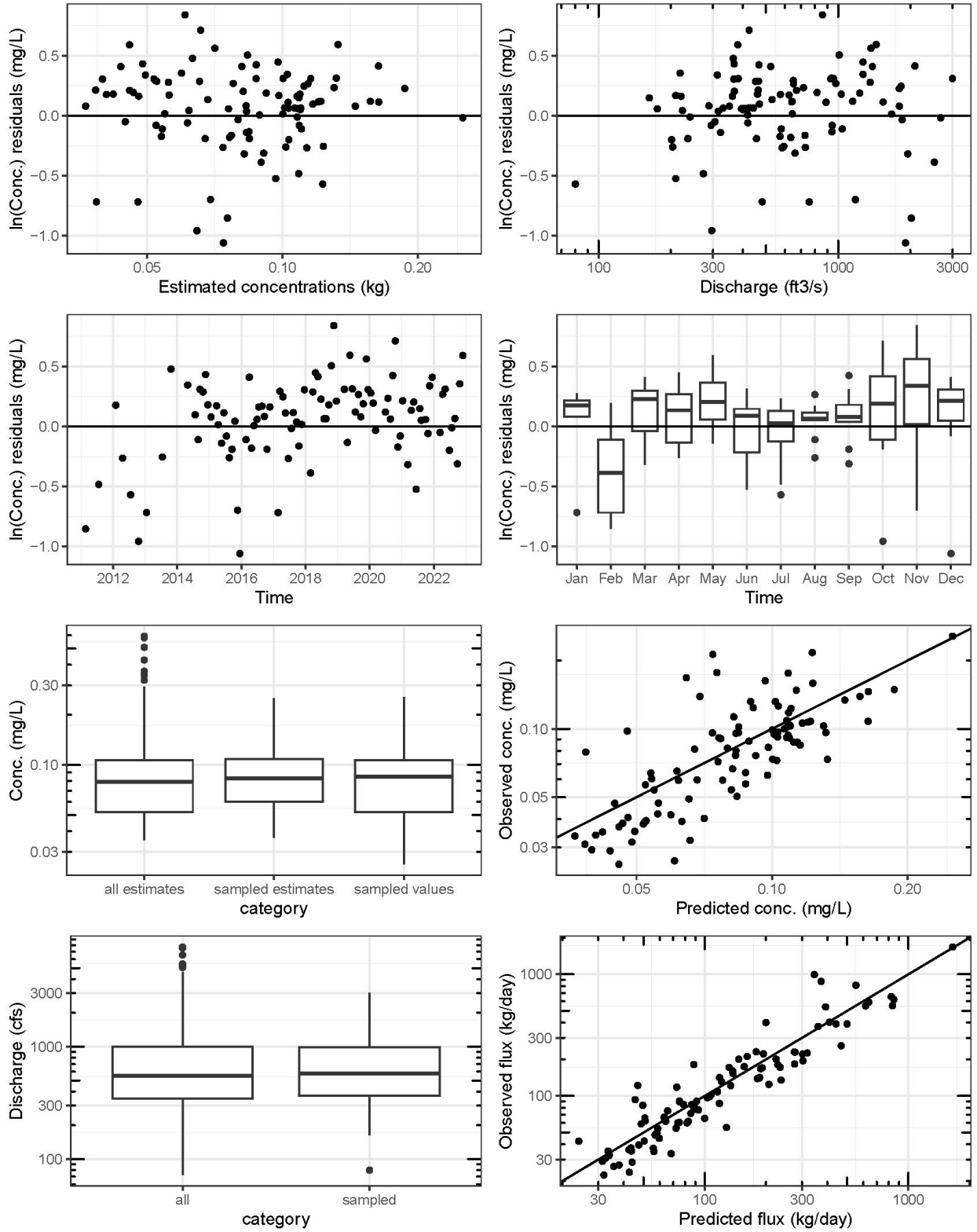
TP, Fox River at Waterford



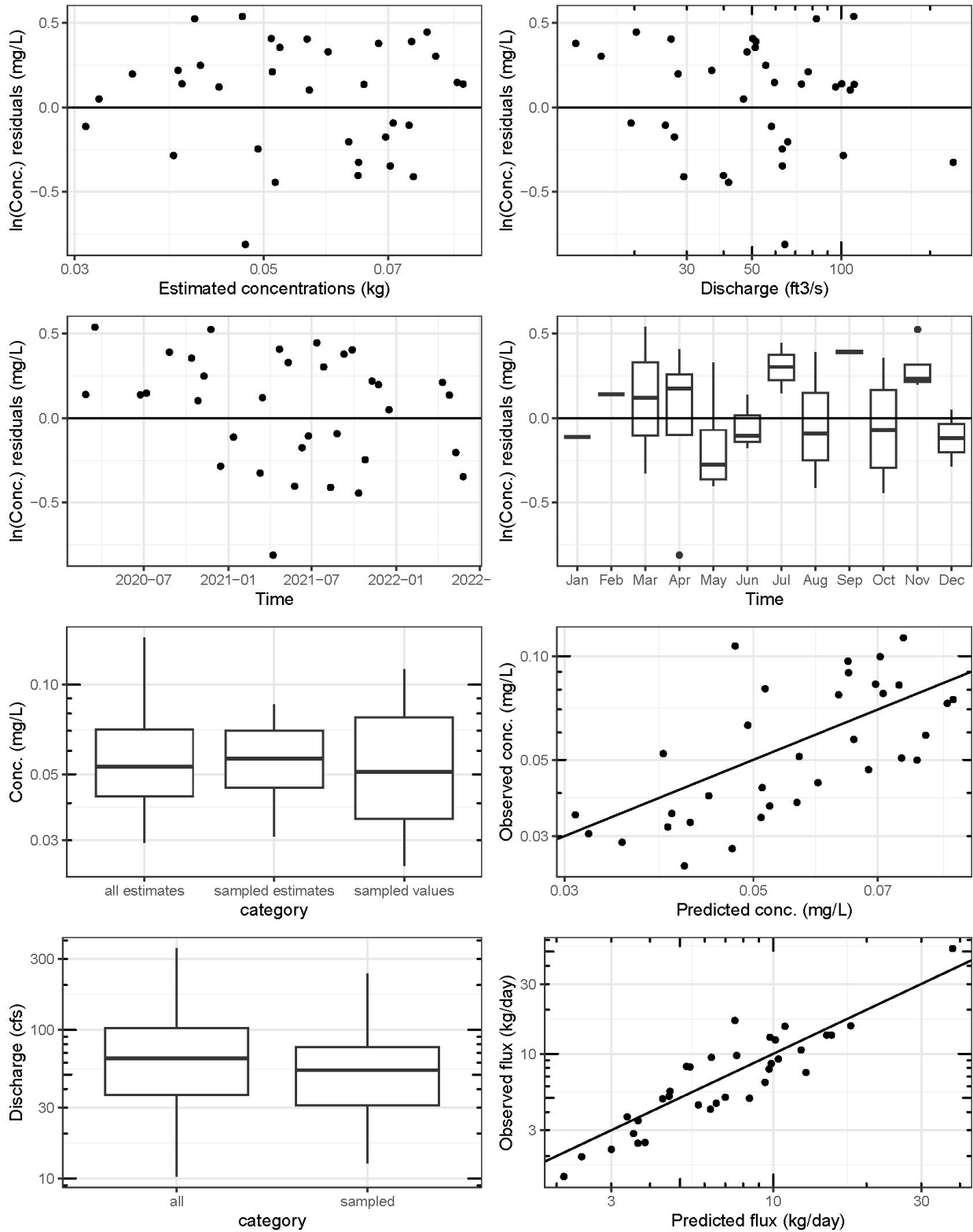
TP, Fox River at Waukesha



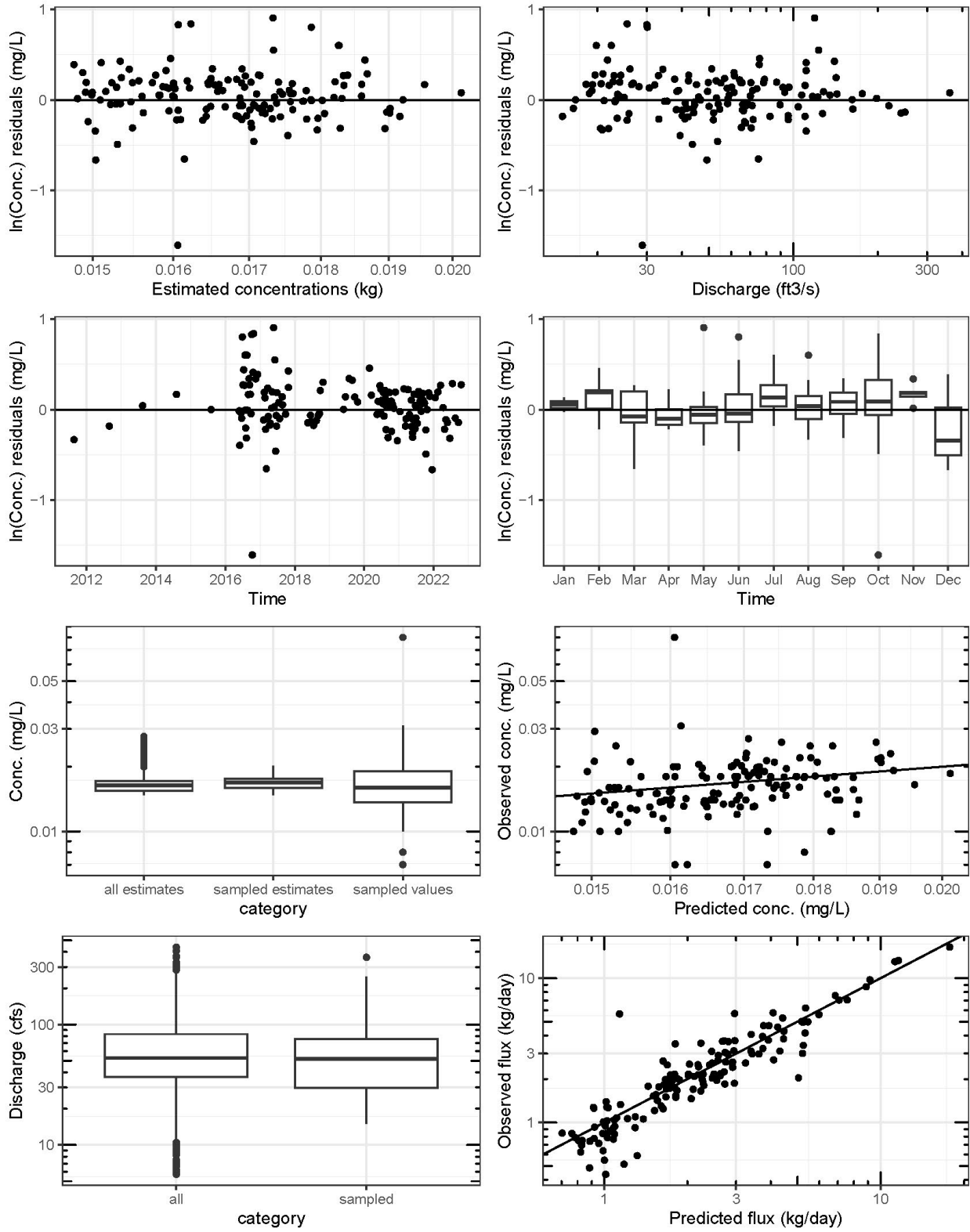
TP, Fox River near New Munster



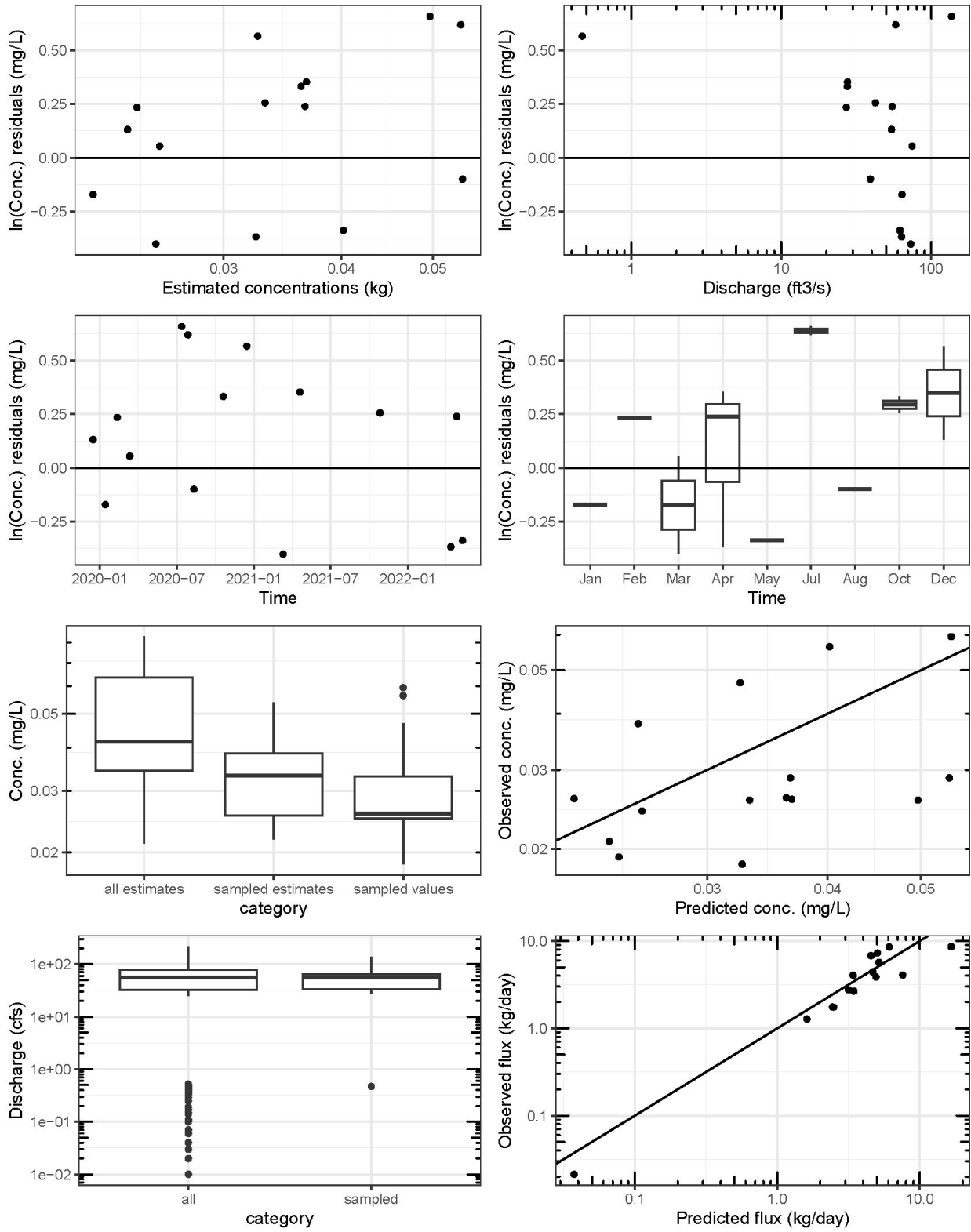
TP, Honey Creek



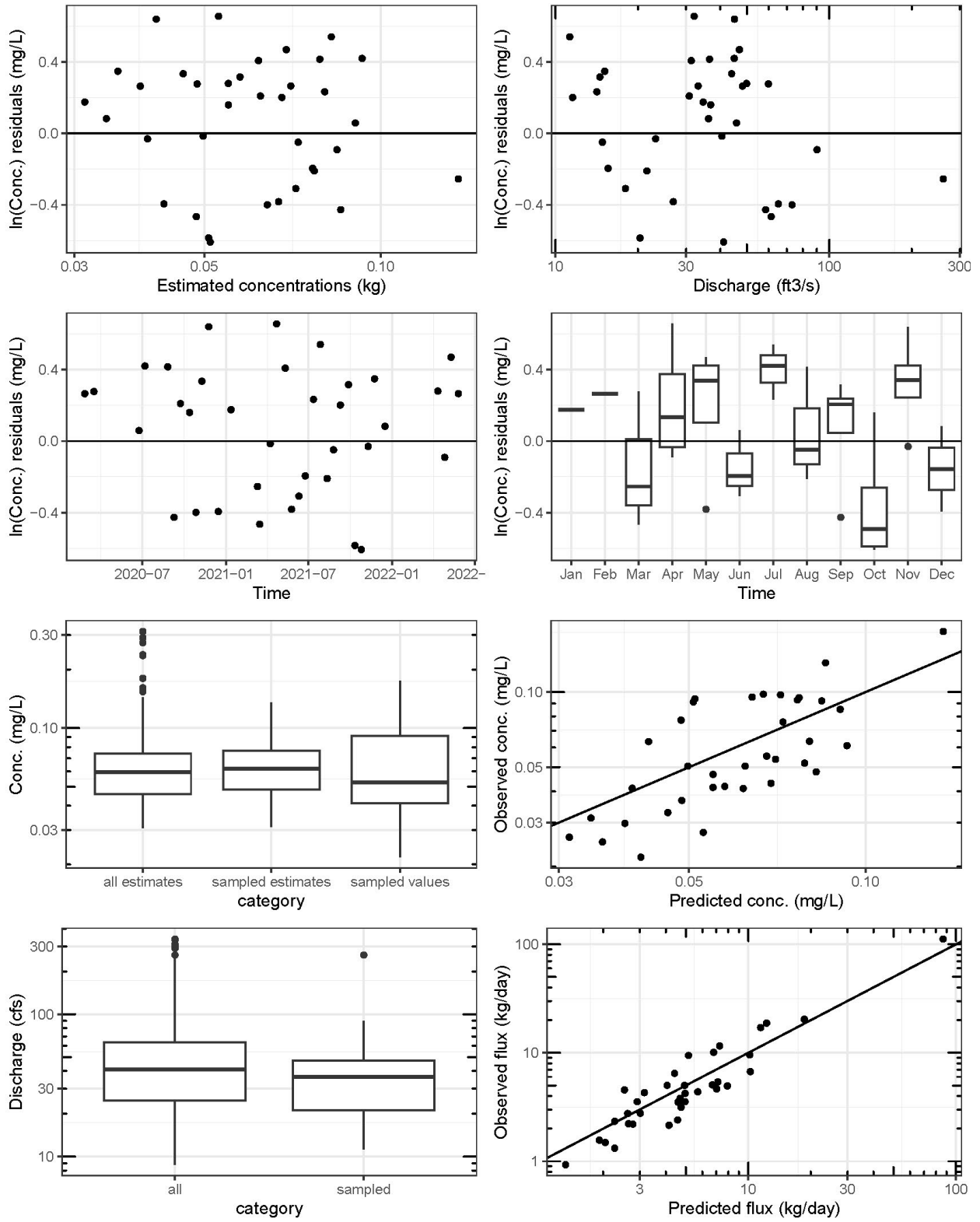
TP, Mukwonago River at Mukwonago



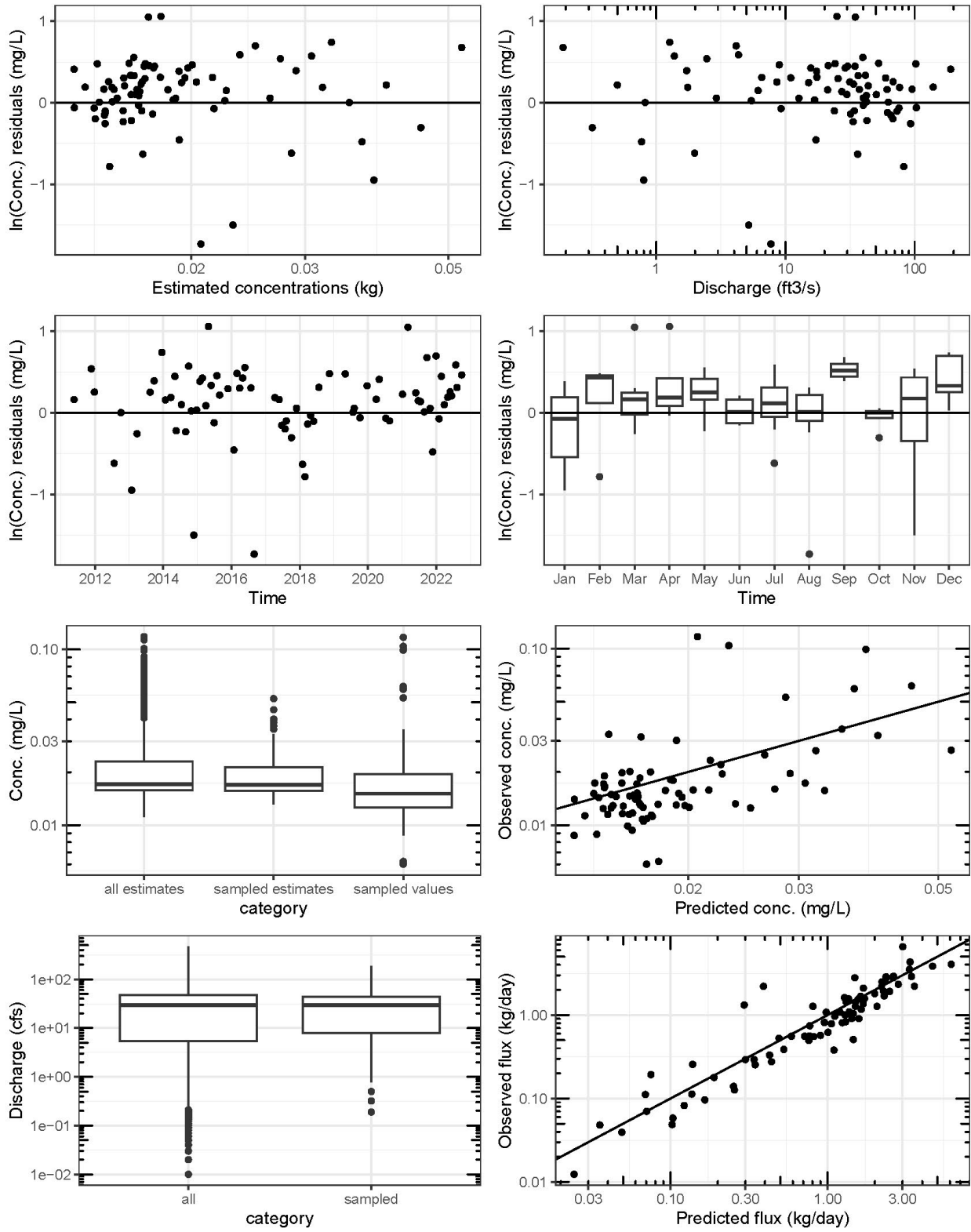
TP, Muskego Lake



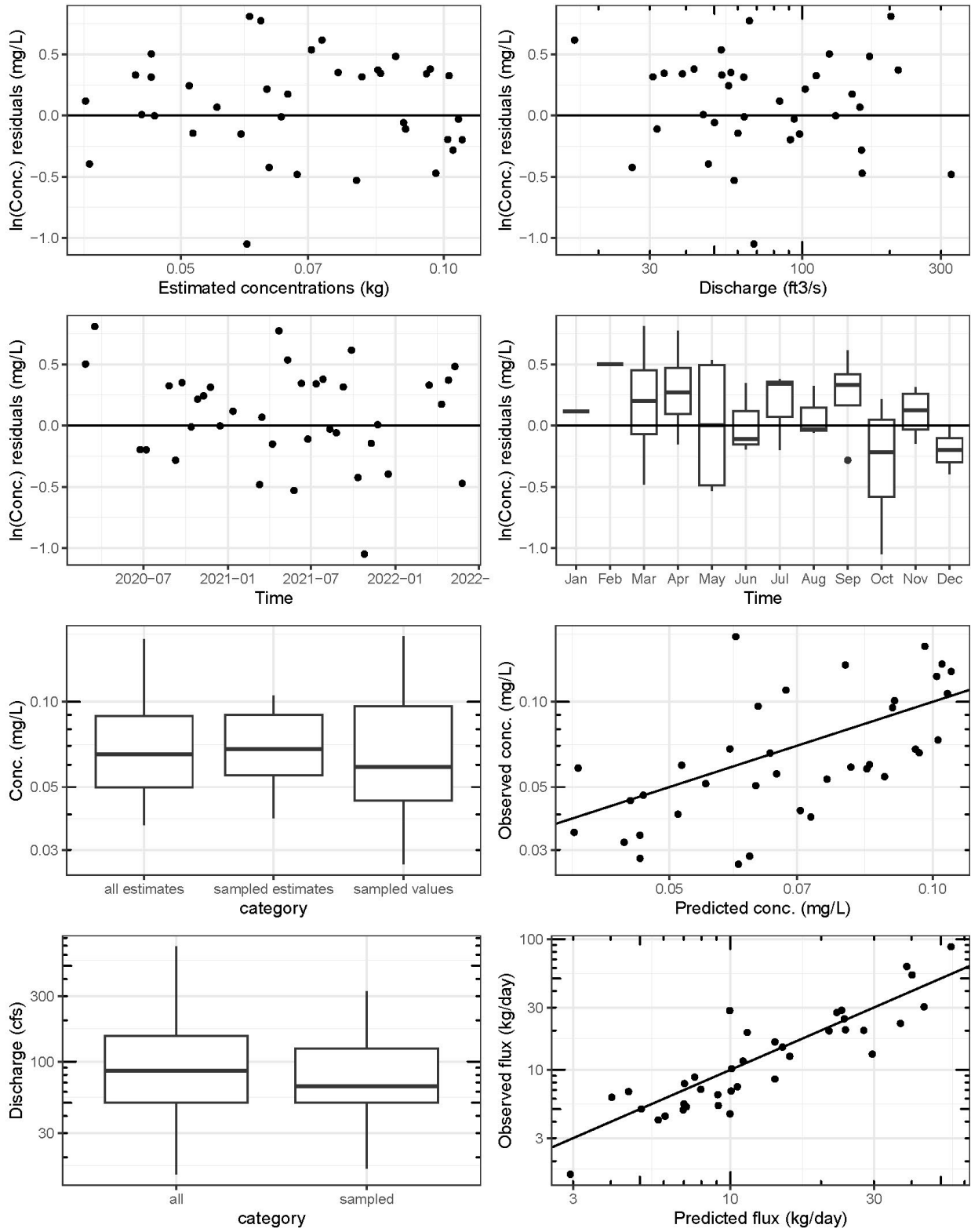
TP, Sugar Creek



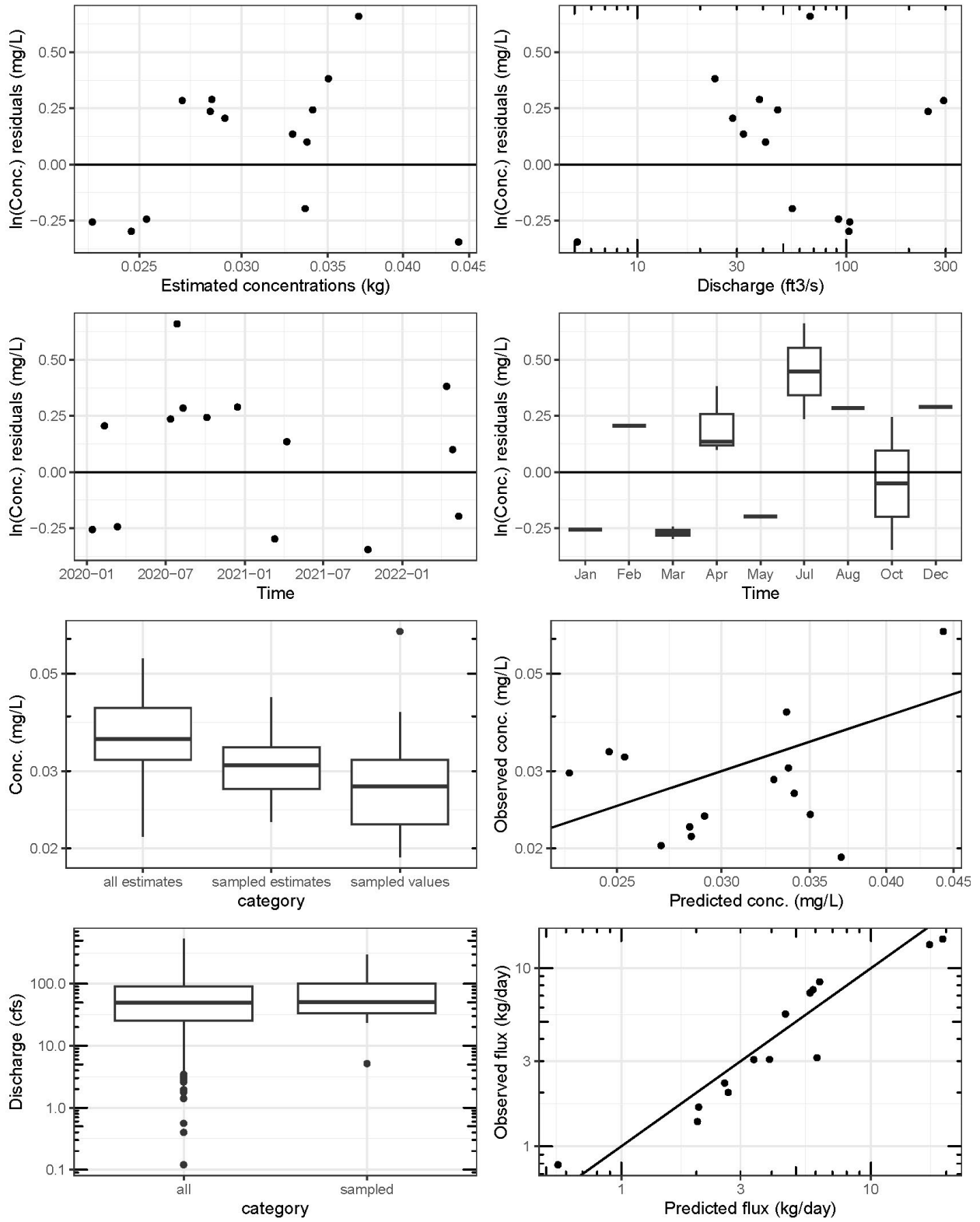
TP, White River at Lake Geneva



TP, White River



TP, Wind Lake

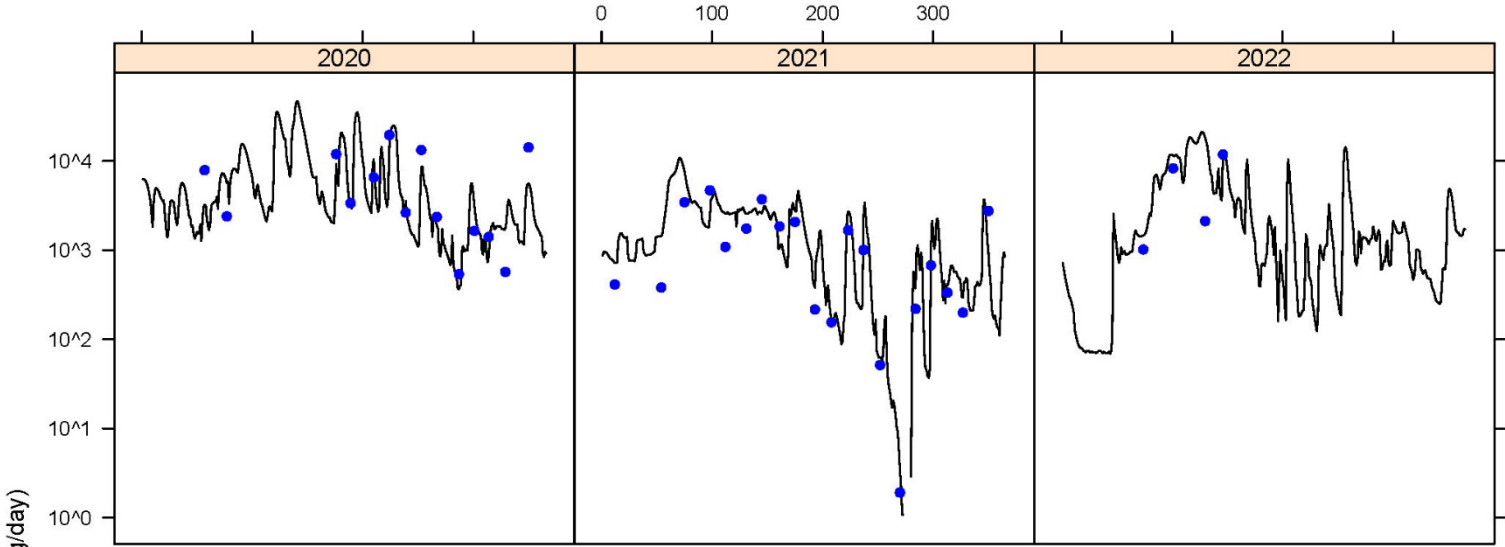


APPENDIX E

TIME SERIES PLOTS FOR TSS LOAD ESTIMATION

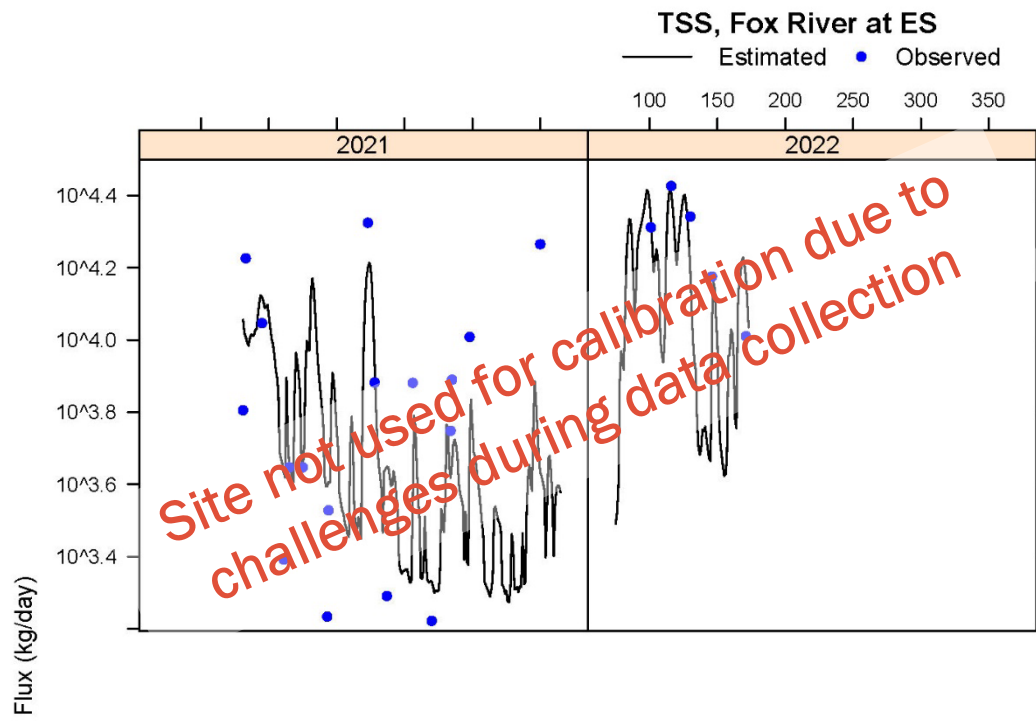
TSS, Des Plaines River at Russell, IL

— Estimated • Observed



DOY

E.1

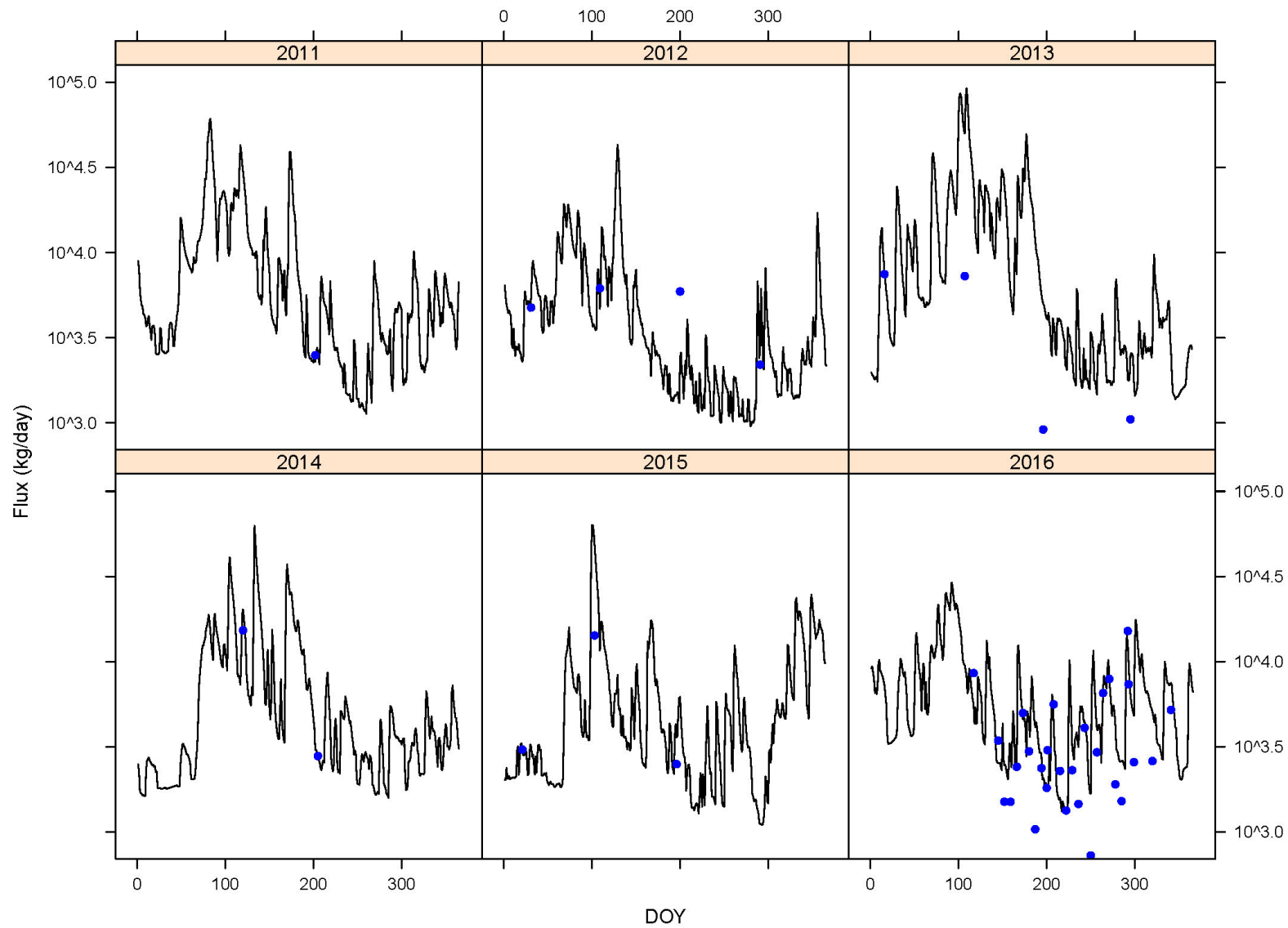


DOY

E.2

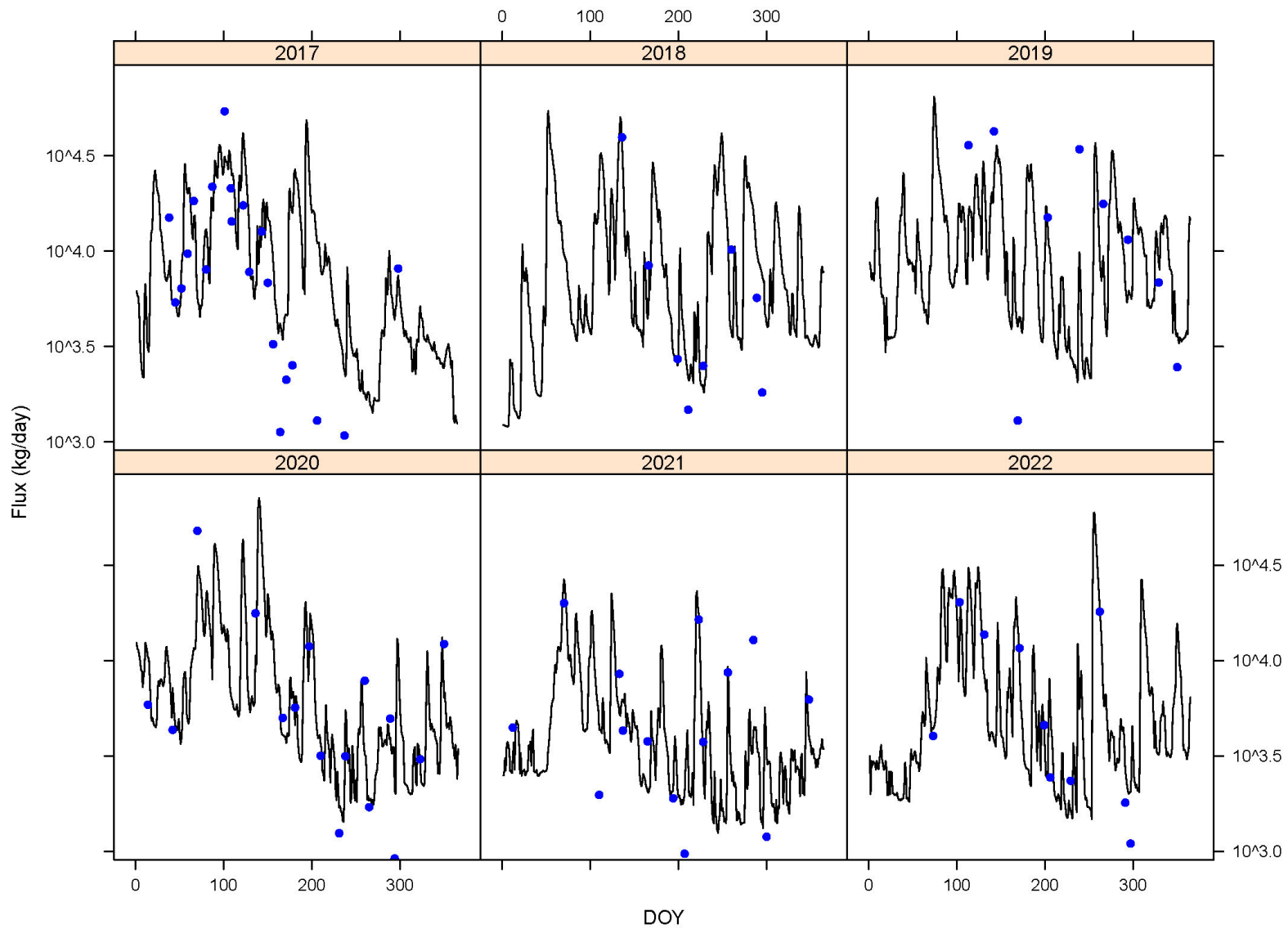
TSS, Fox River at I

— Estimated • Observed



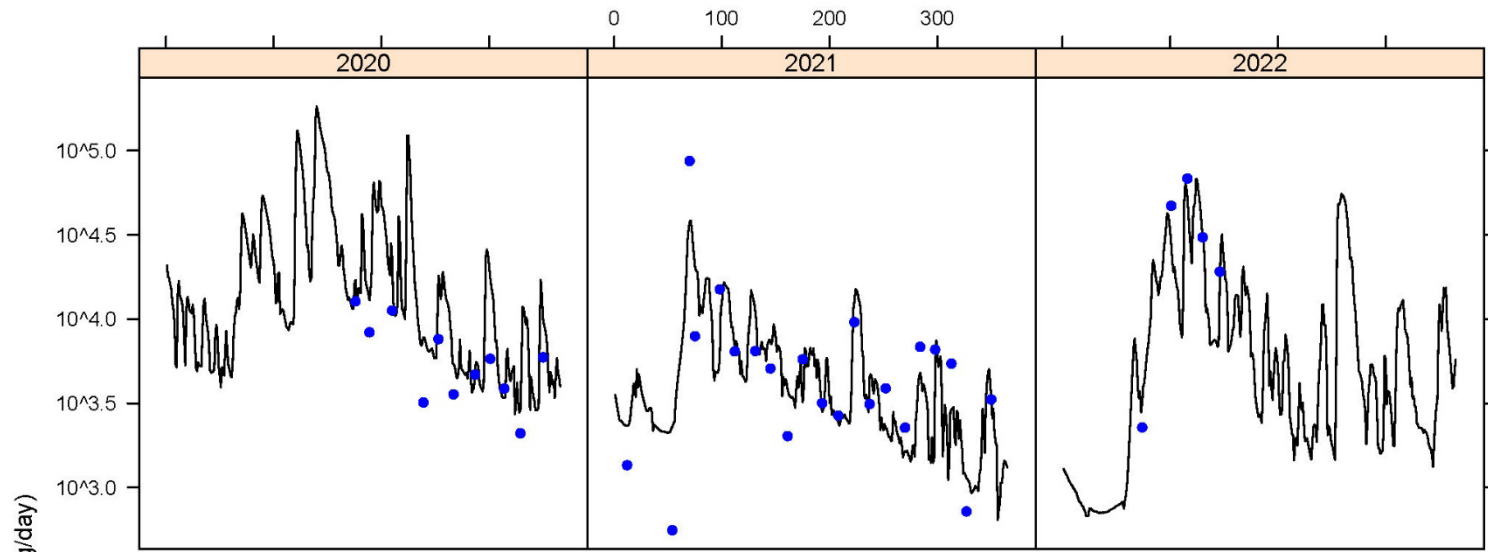
TSS, Fox River at I

— Estimated • Observed



TSS, Fox River at Rochester

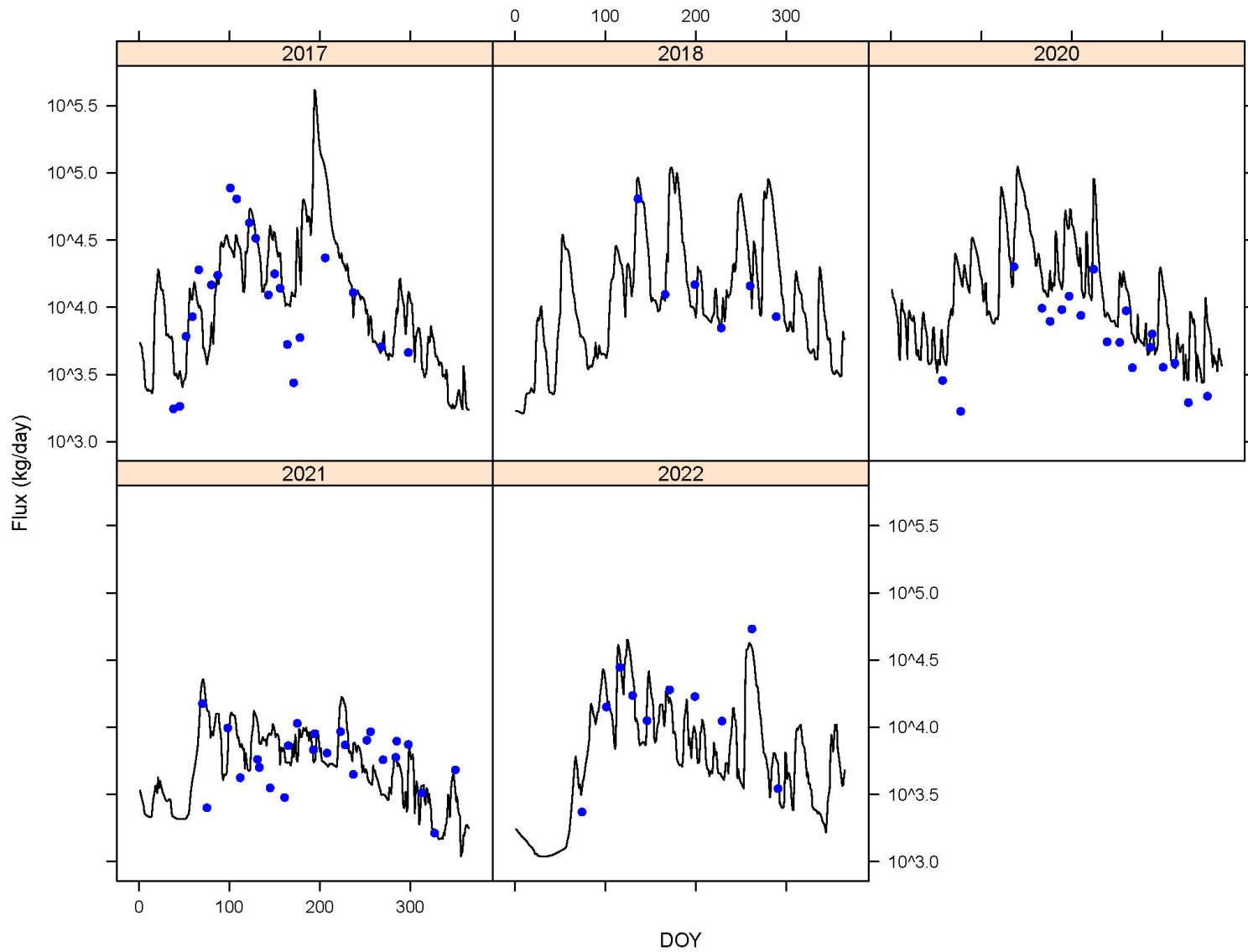
— Estimated • Observed



DOY

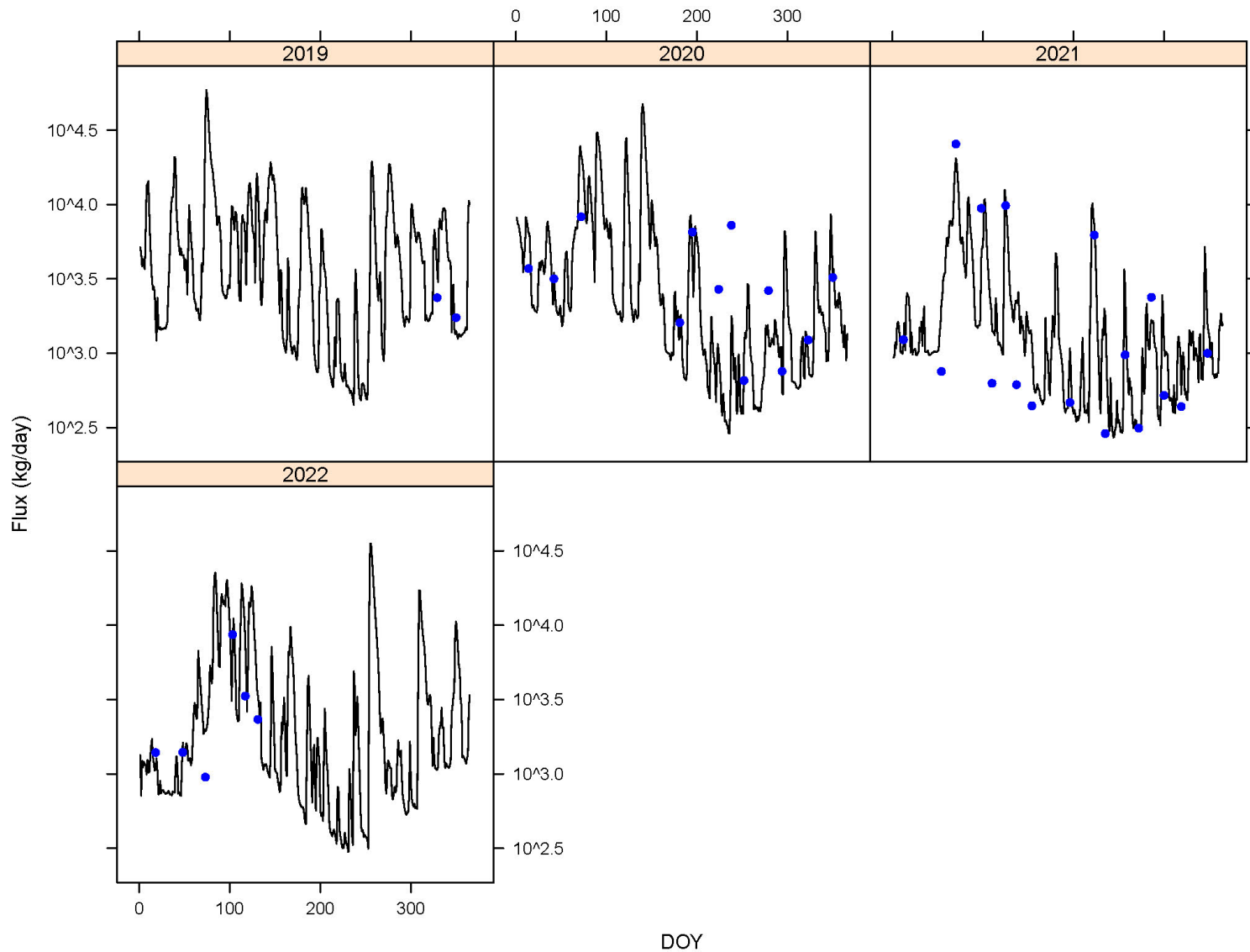
TSS, Fox River at Waterford

— Estimated • Observed



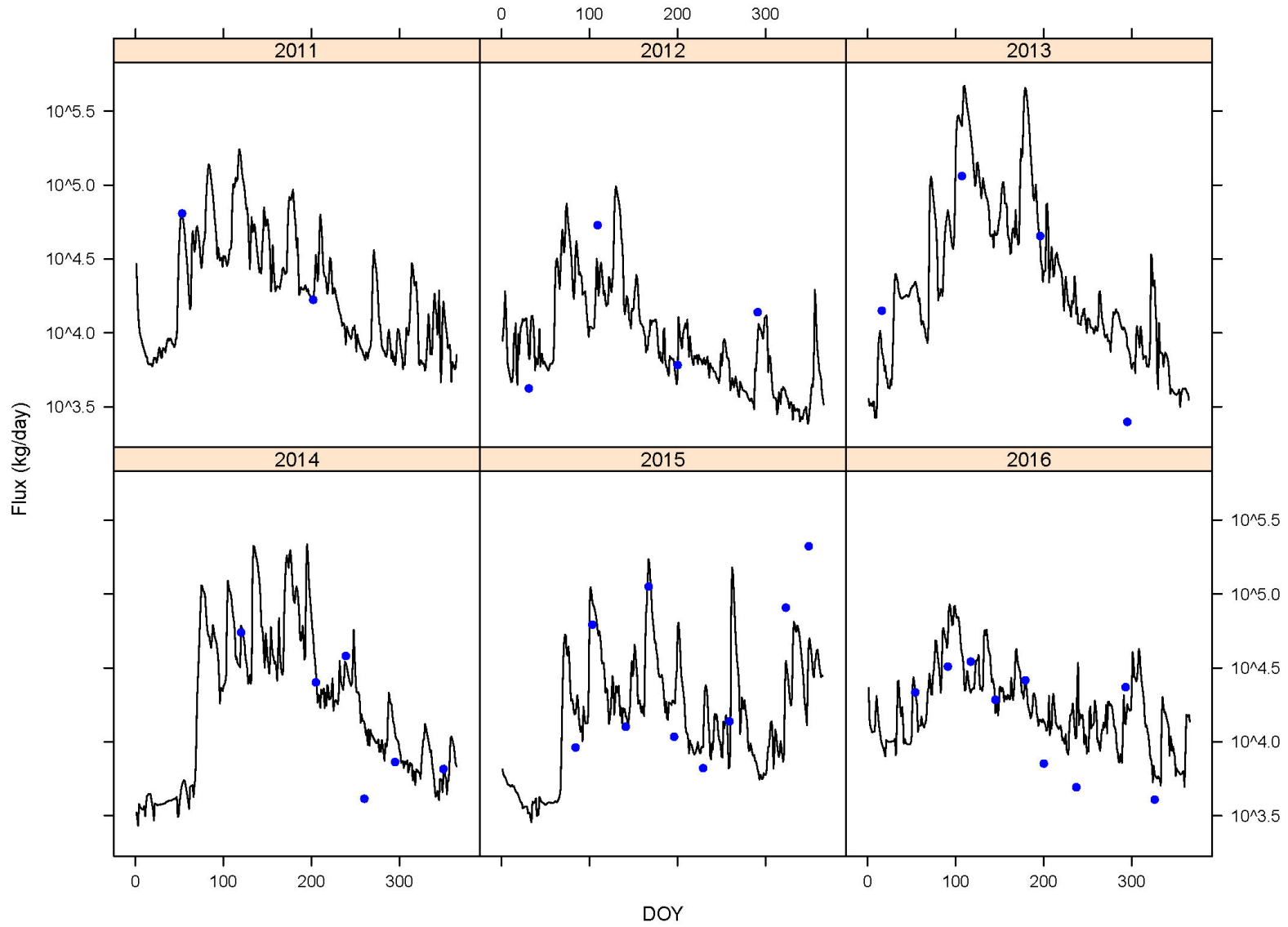
TSS, Fox River at Waukesha

— Estimated • Observed



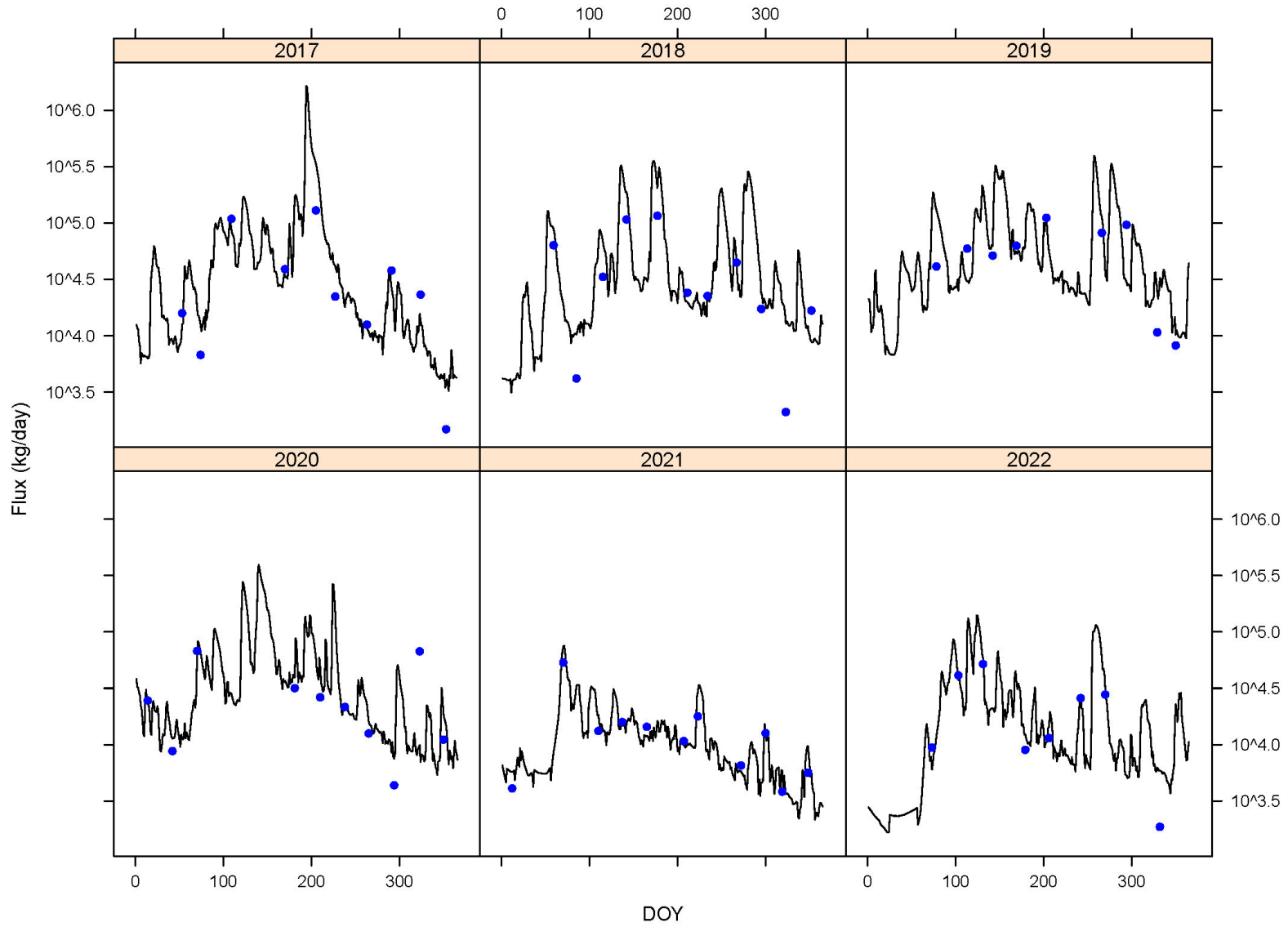
TSS, Fox River near New Munster

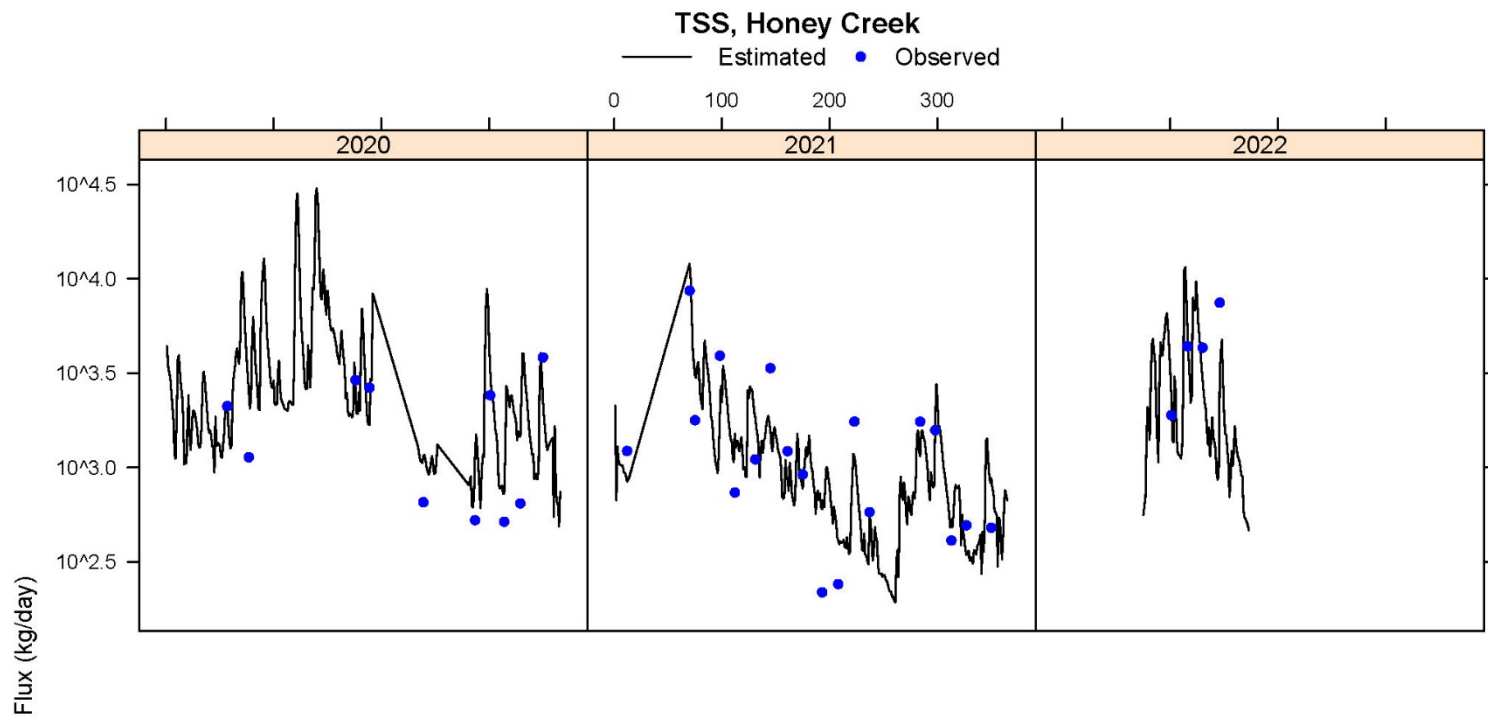
— Estimated • Observed



TSS, Fox River near New Munster

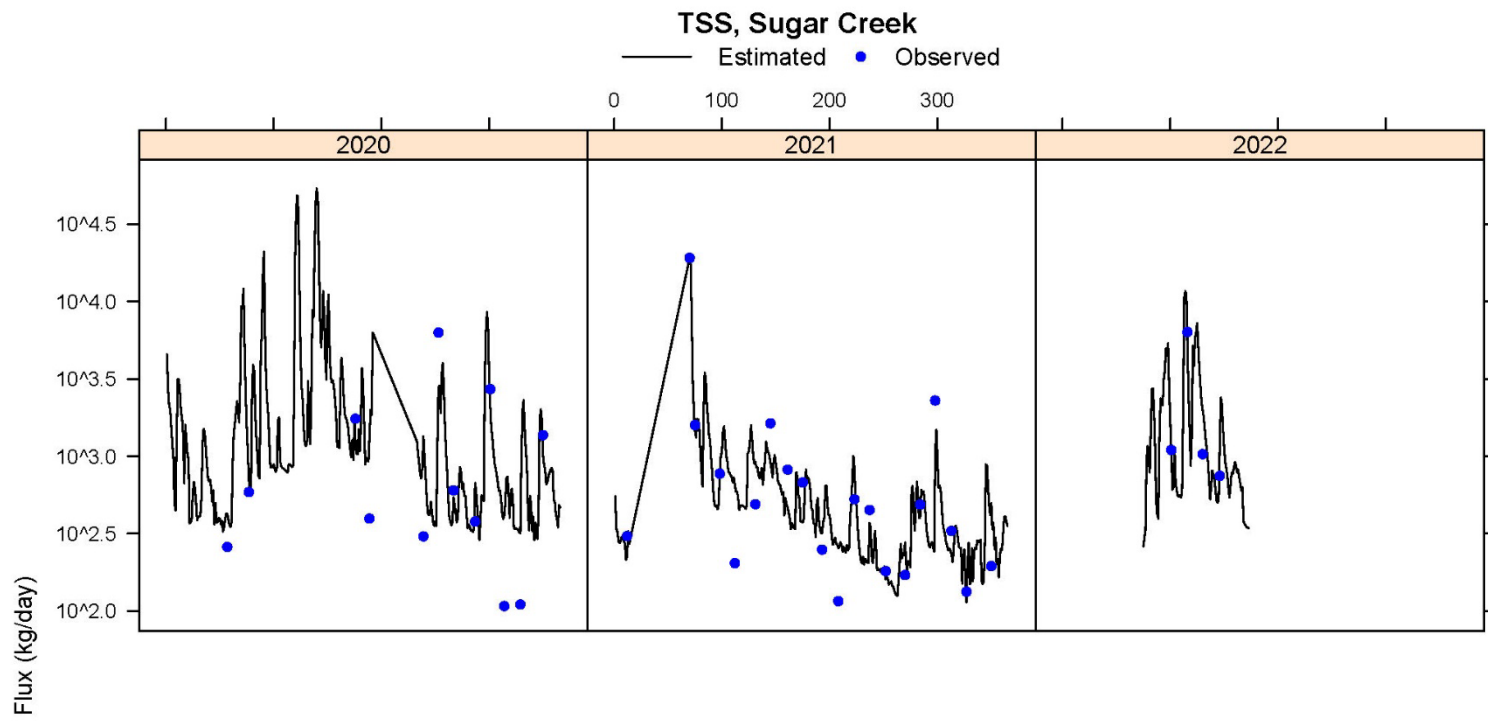
— Estimated • Observed

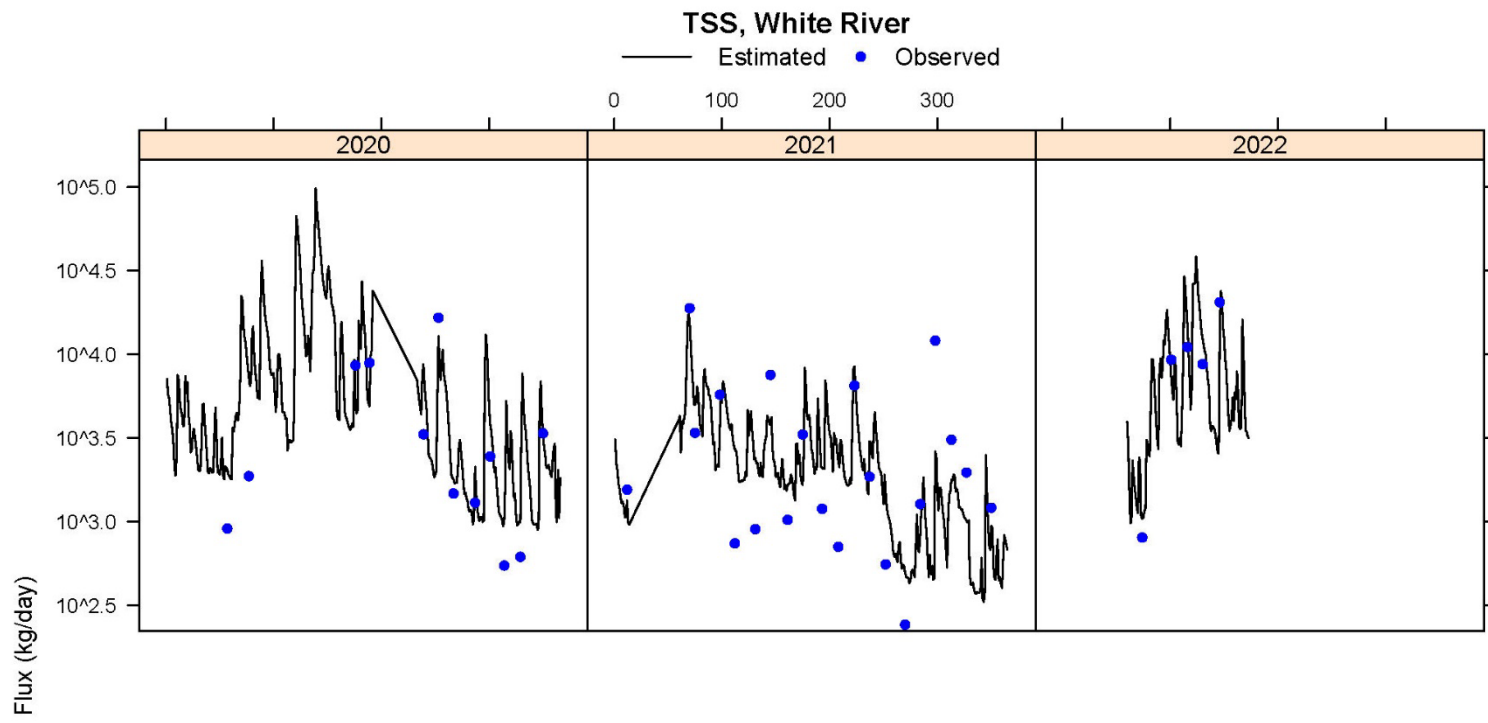




DOY

E.10

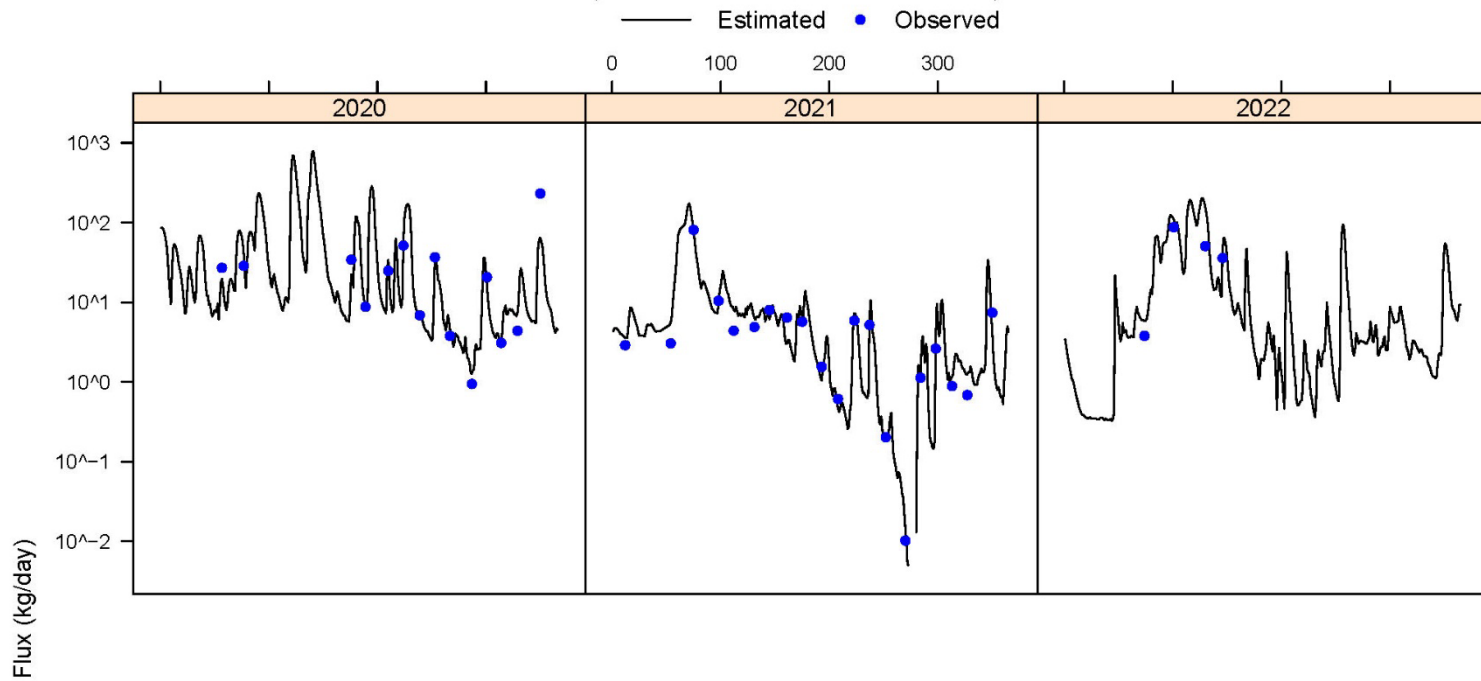




APPENDIX F

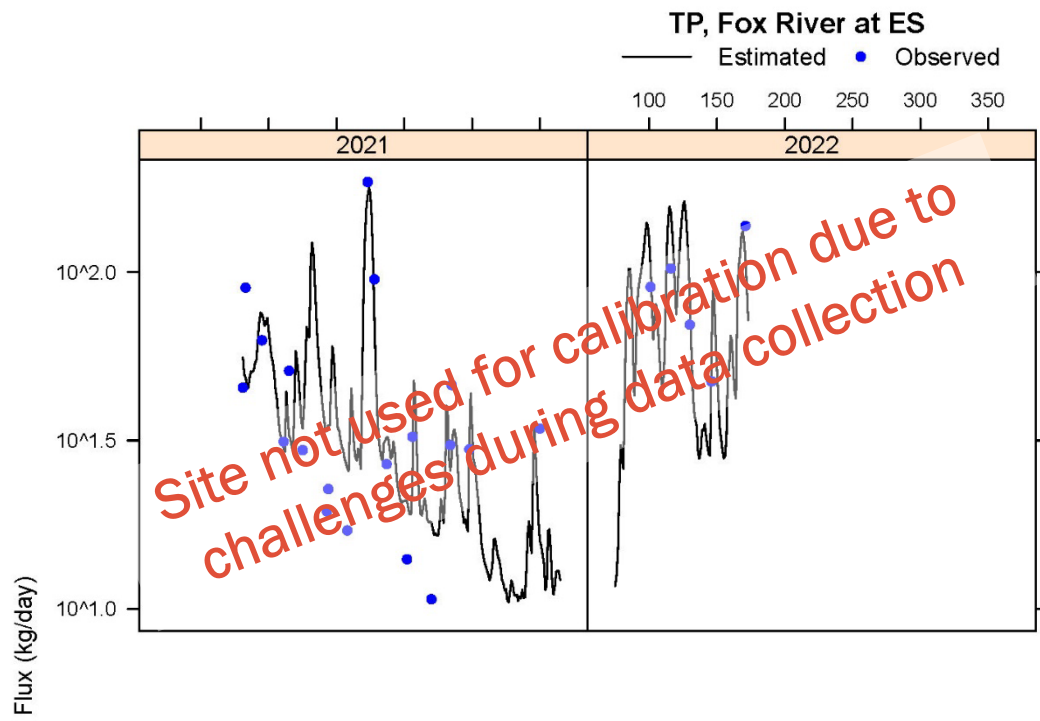
TIME SERIES PLOTS FOR TP LOAD ESTIMATION

TP, Des Plaines River at Russell, IL



DOY

F.1

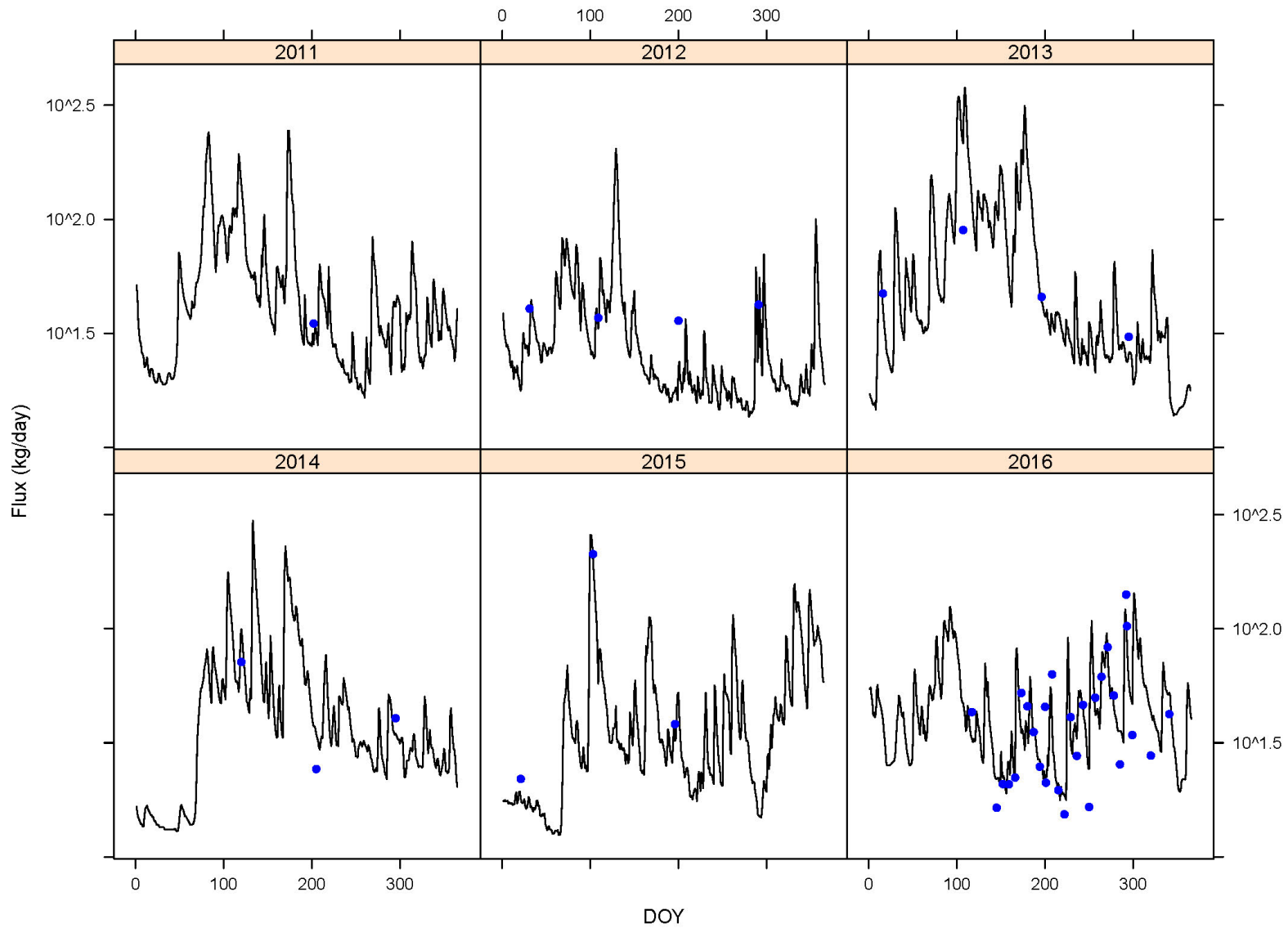


DOY

F.2

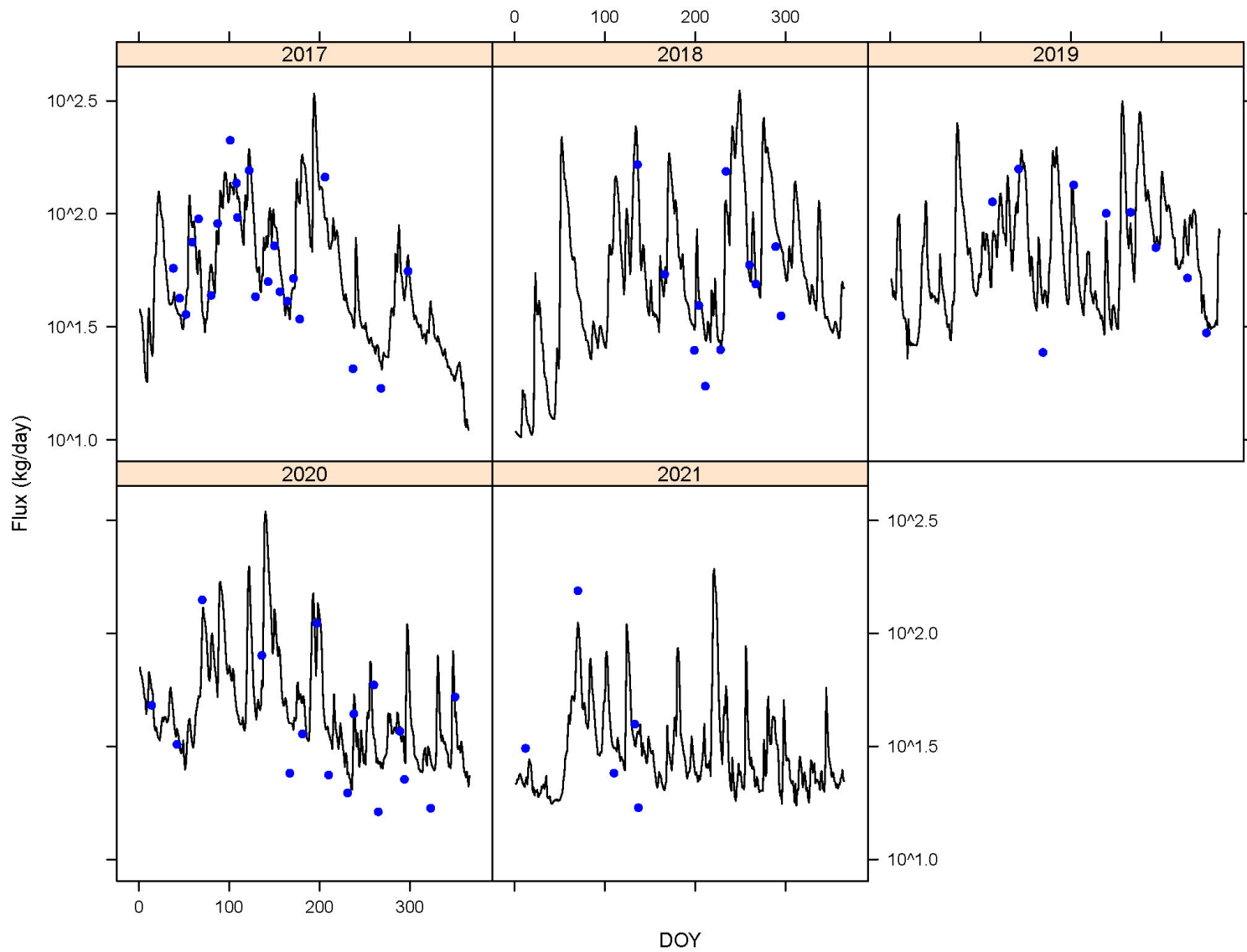
TP, Fox River at I

— Estimated • Observed



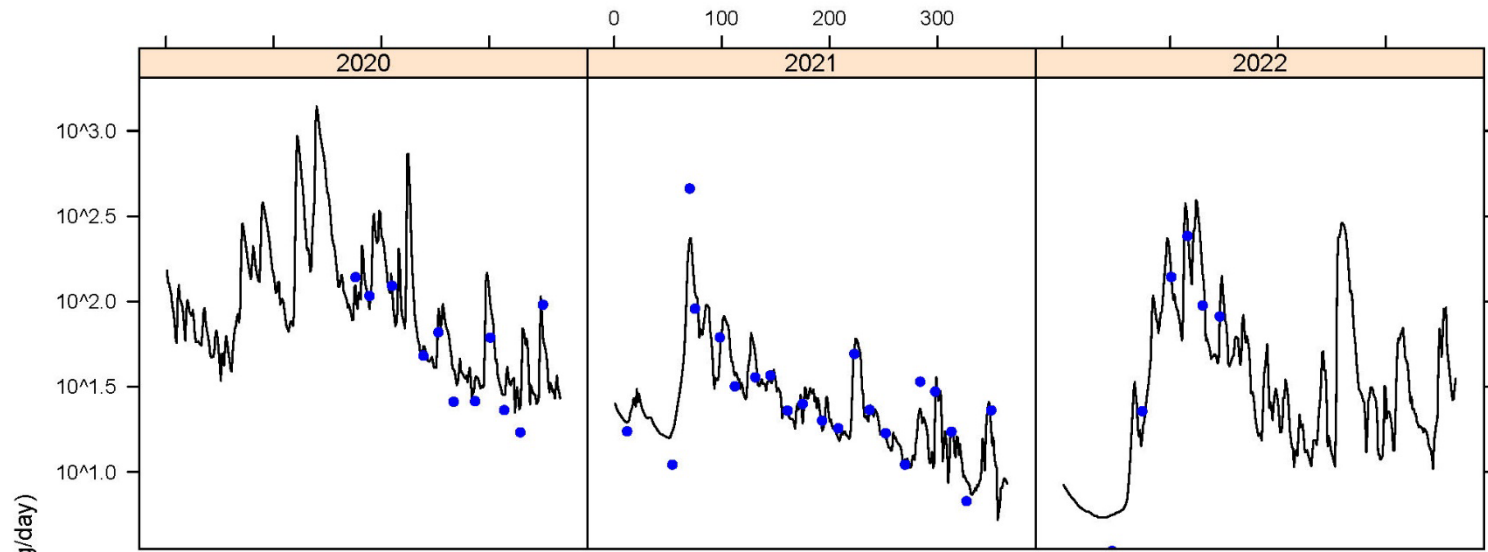
TP, Fox River at I

— Estimated • Observed



TP, Fox River at Rochester

— Estimated • Observed

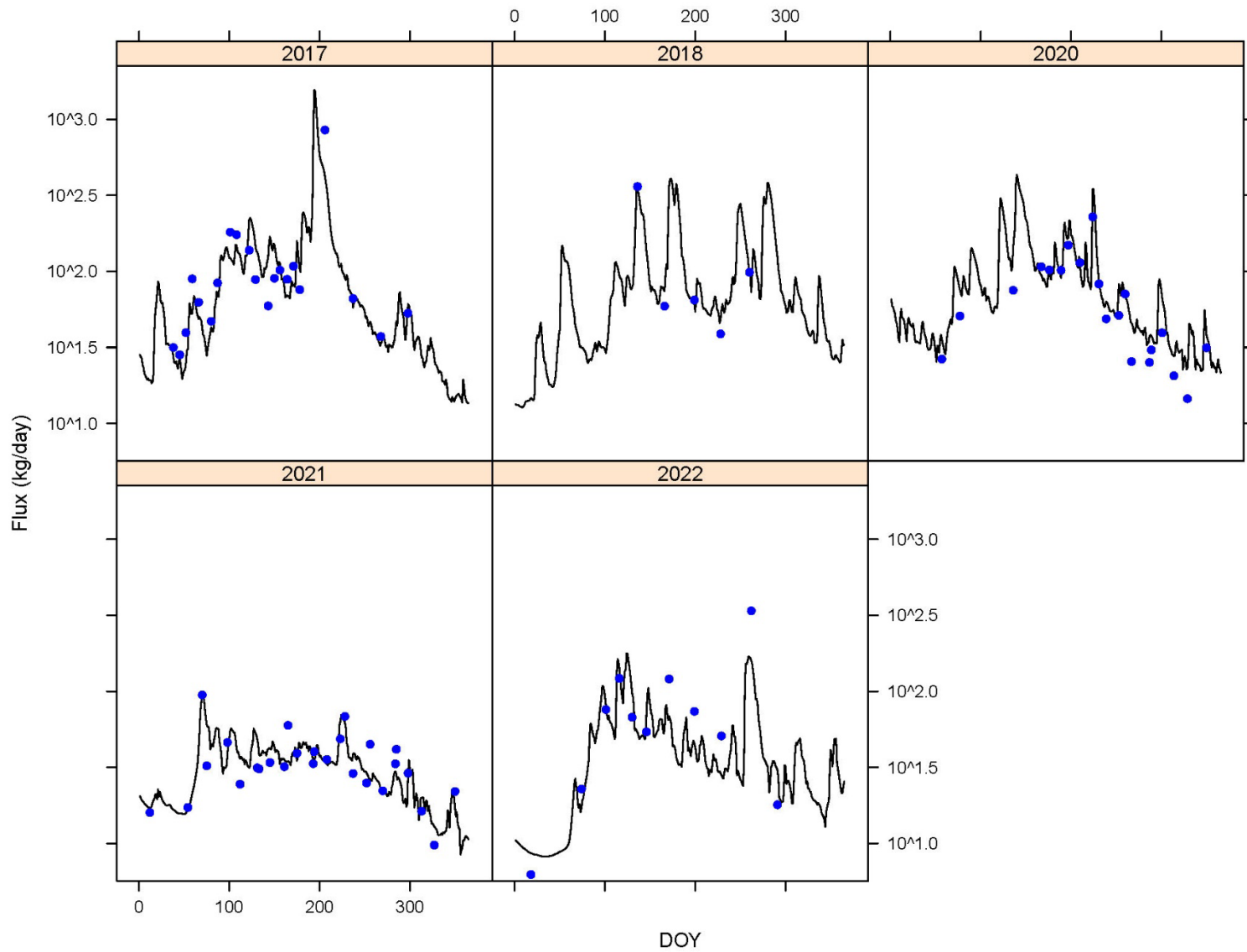


DOY

F.5

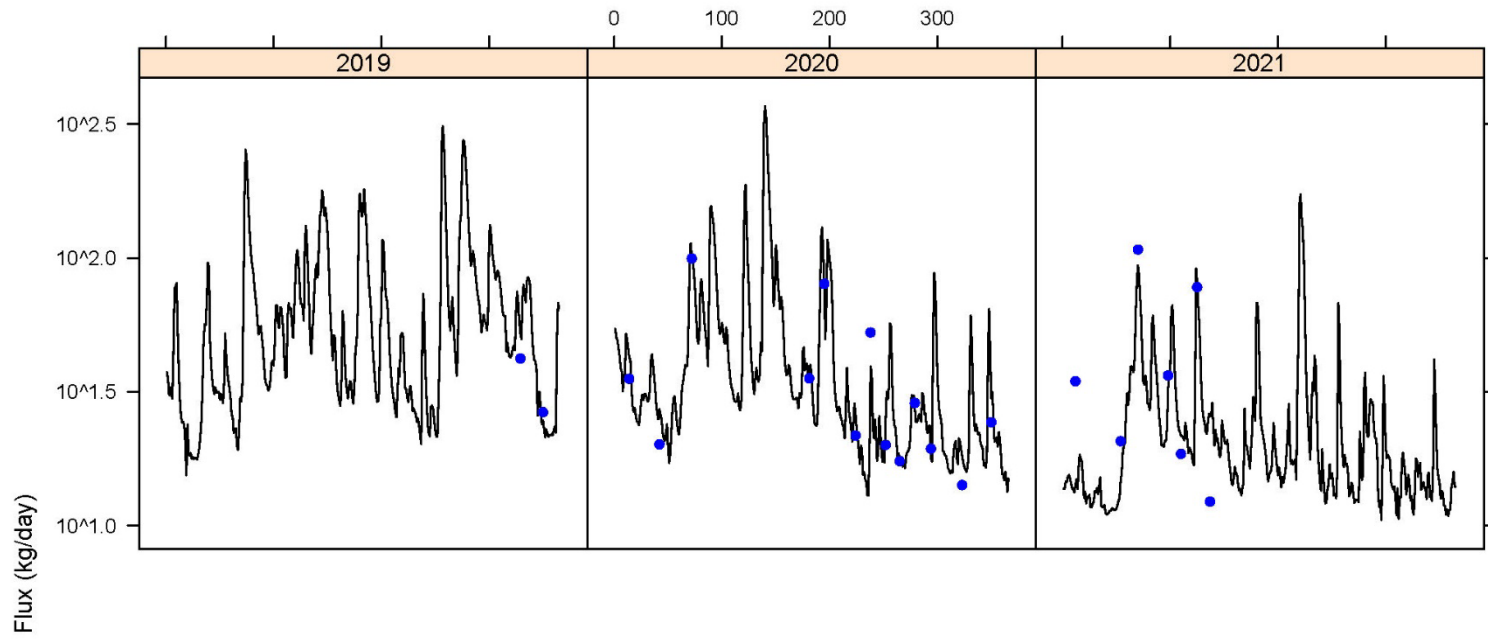
TP, Fox River at Waterford

— Estimated • Observed



TP, Fox River at Waukesha

— Estimated • Observed

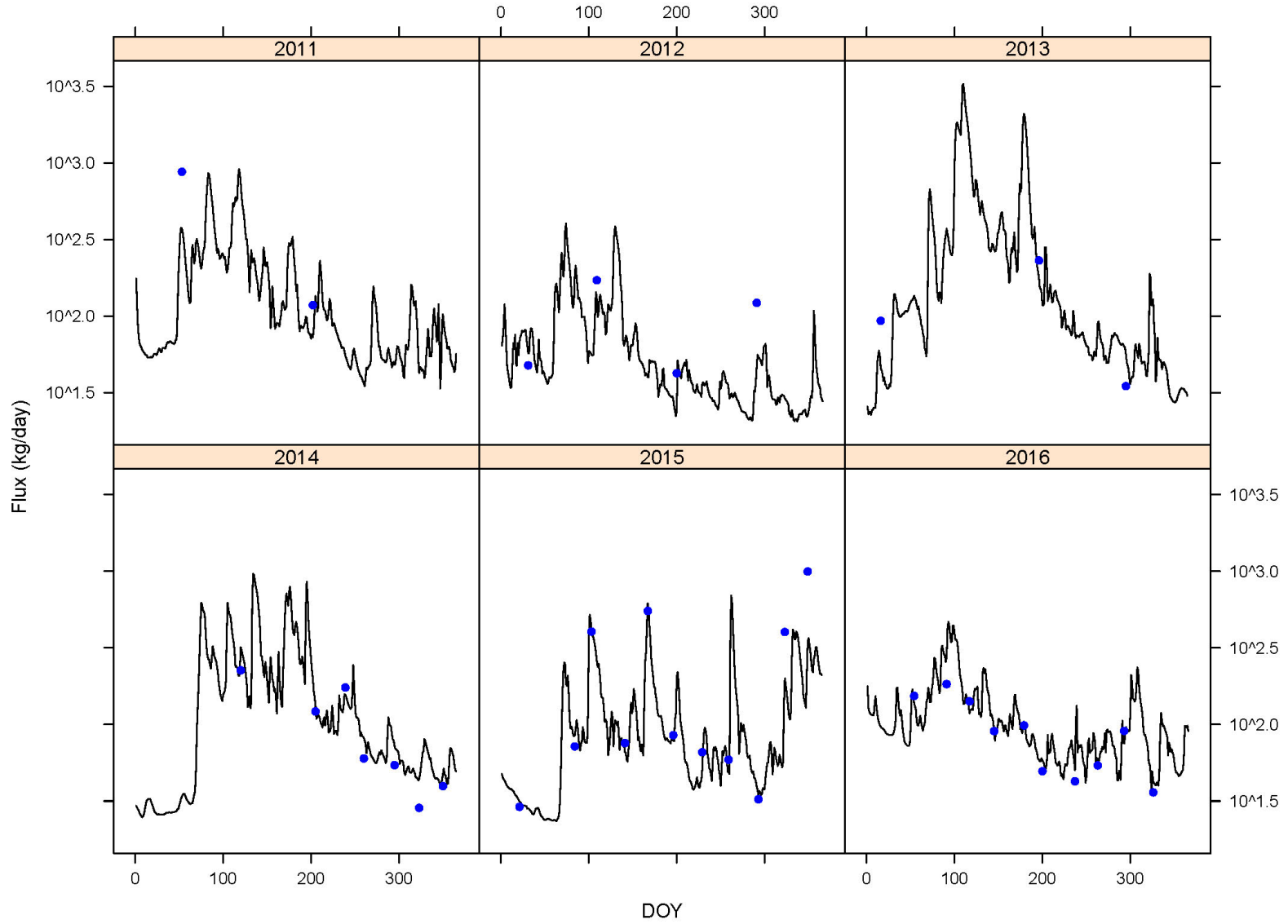


DOY

F.7

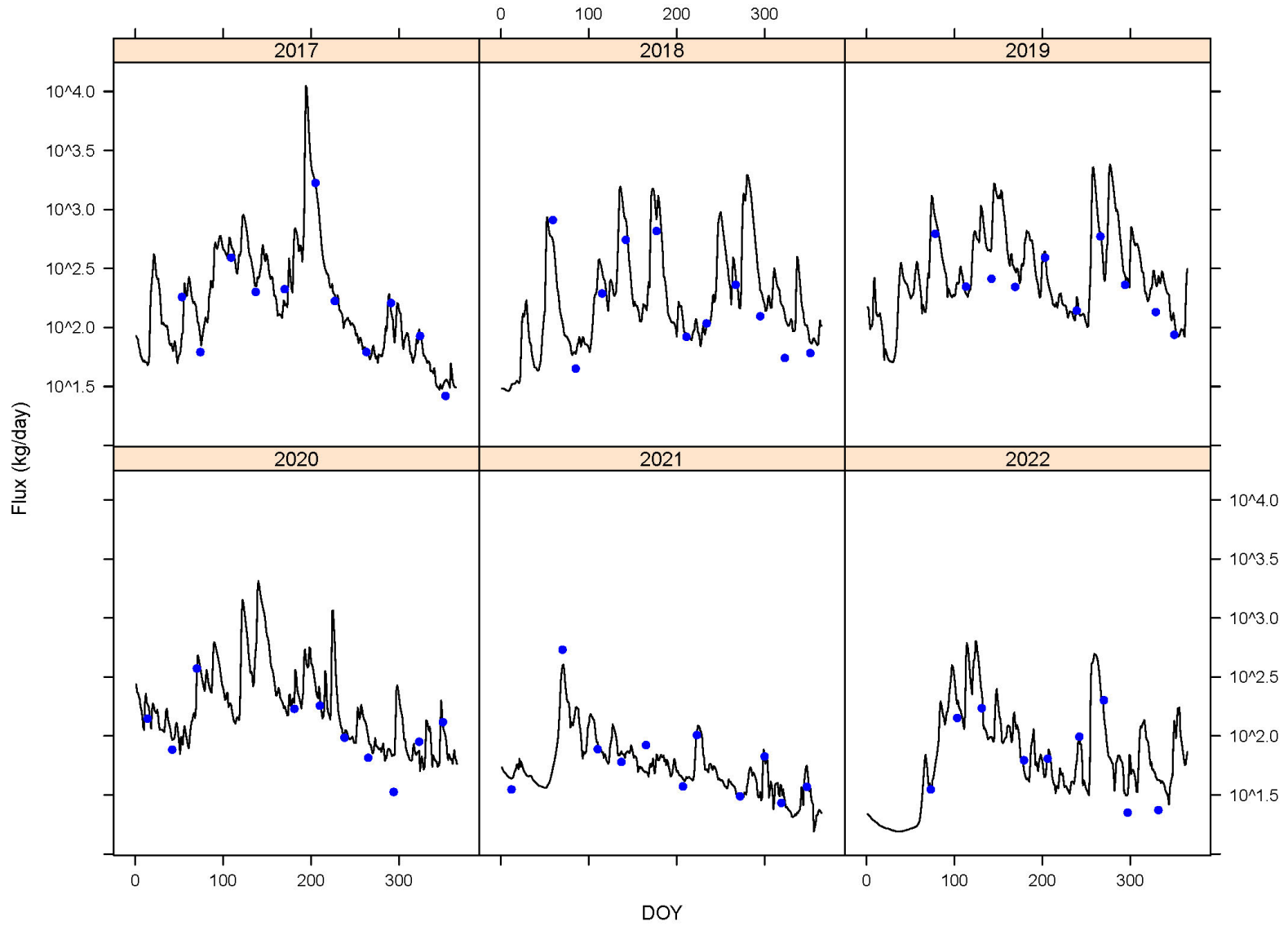
TP, Fox River near New Munster

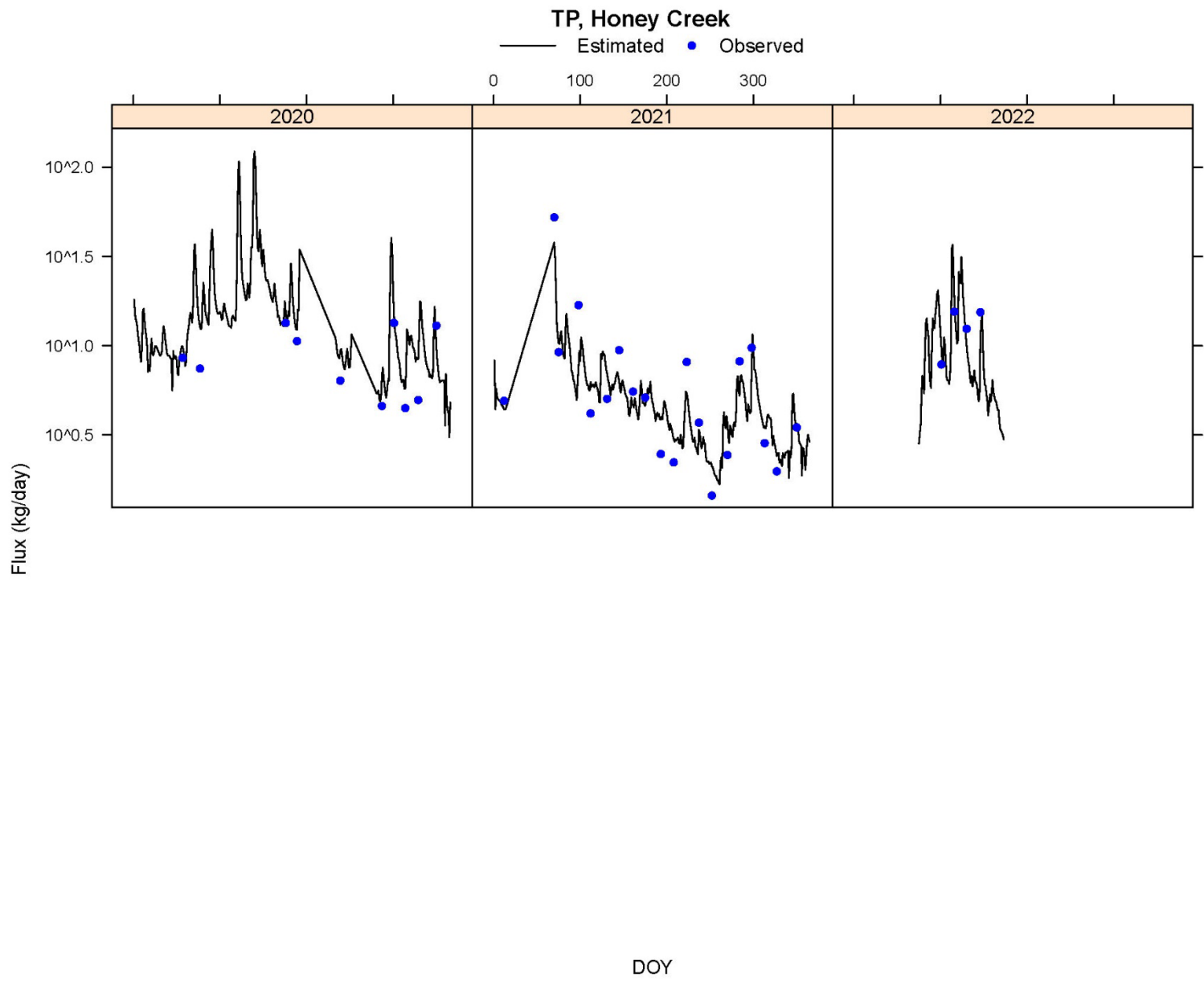
— Estimated • Observed



TP, Fox River near New Munster

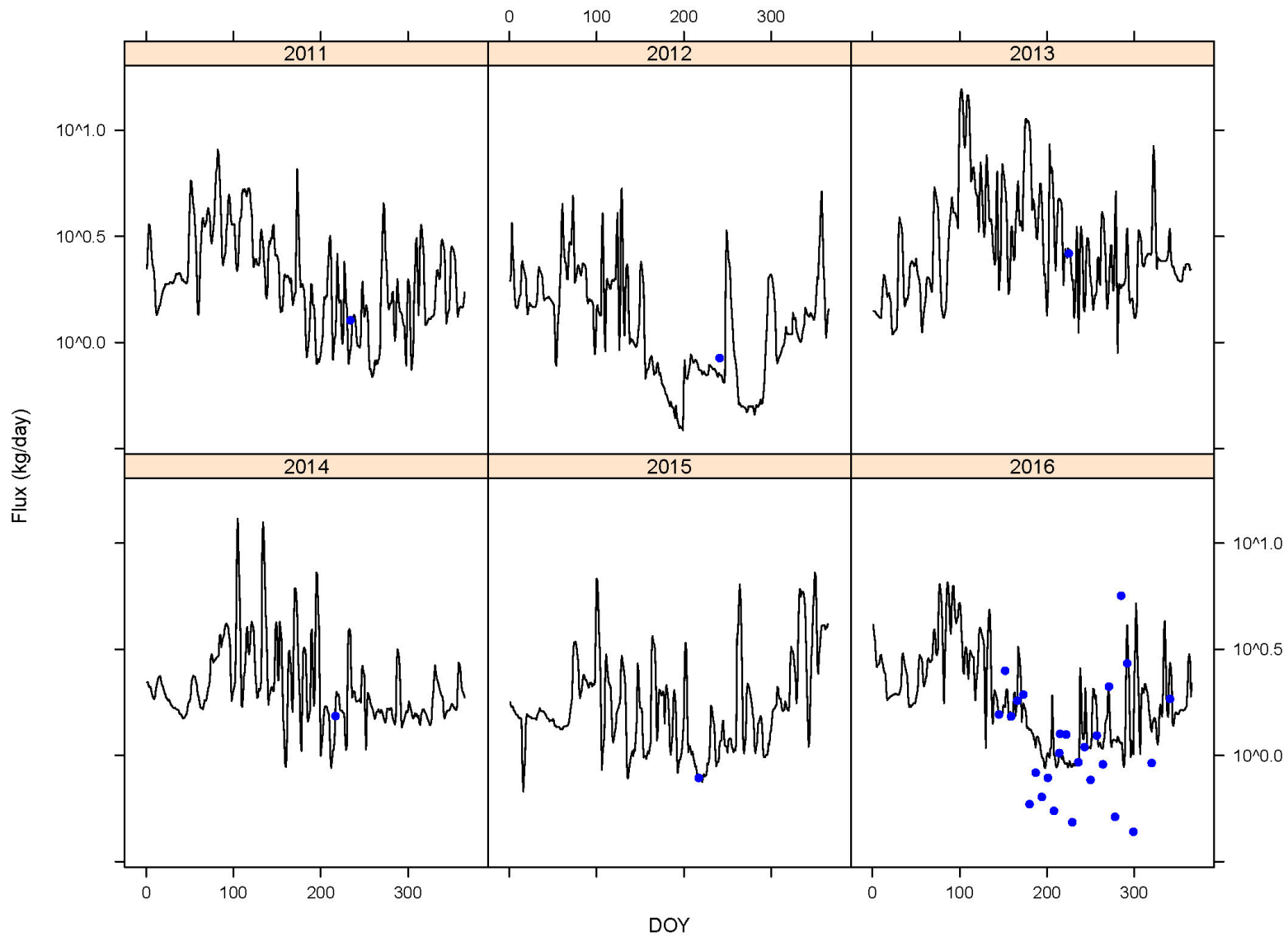
— Estimated • Observed





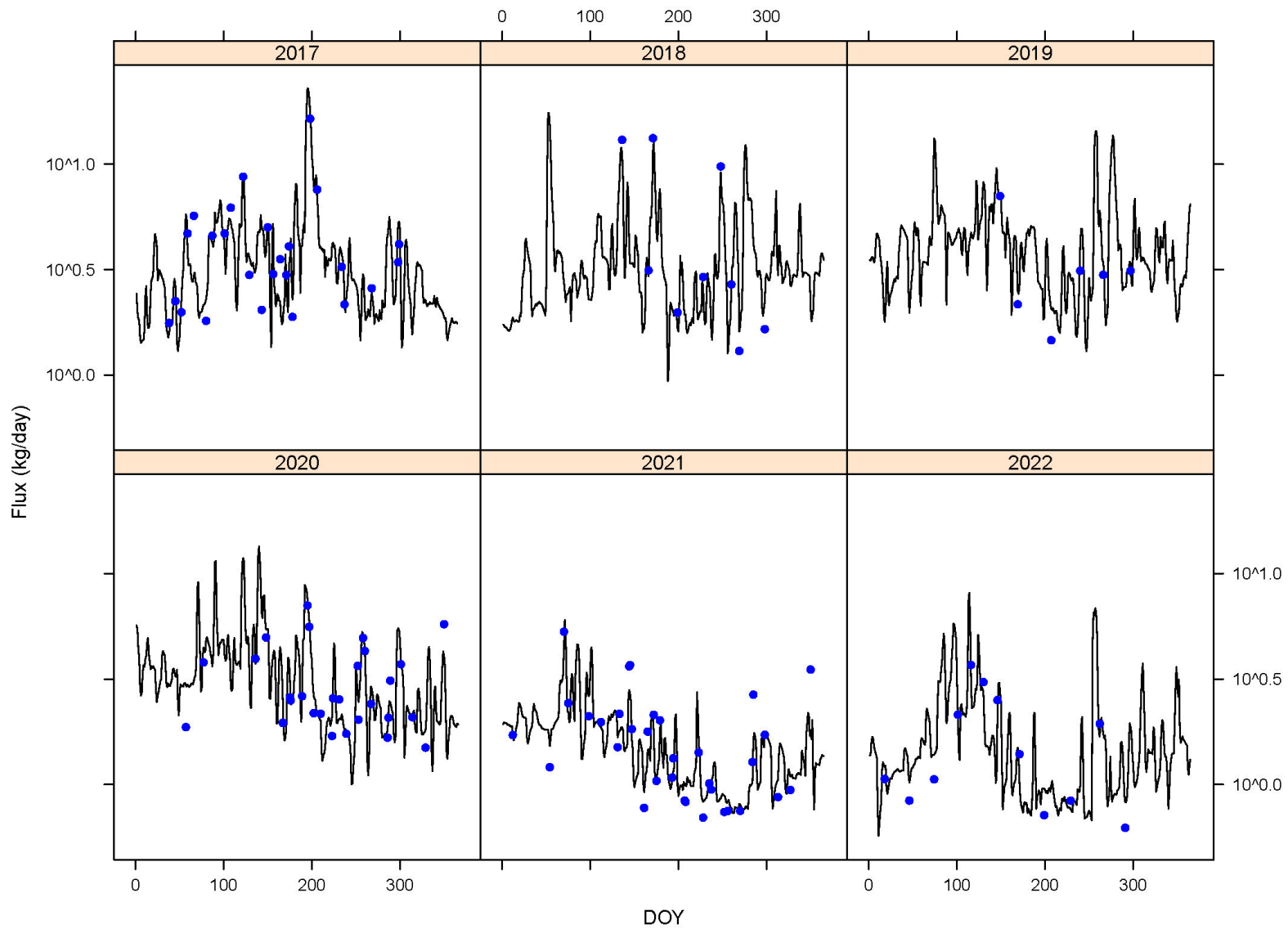
TP, Mukwonago River at Mukwonago

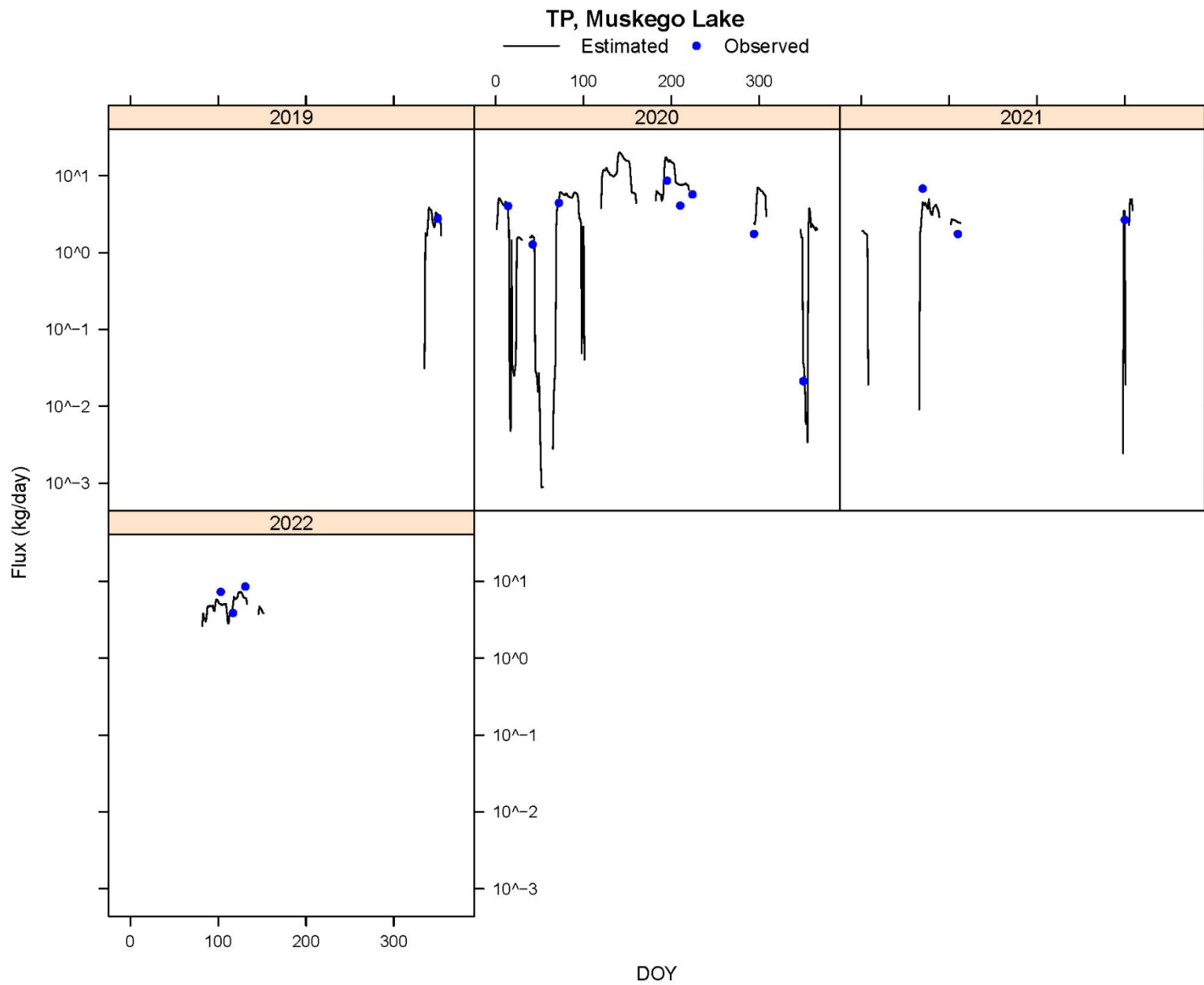
— Estimated • Observed

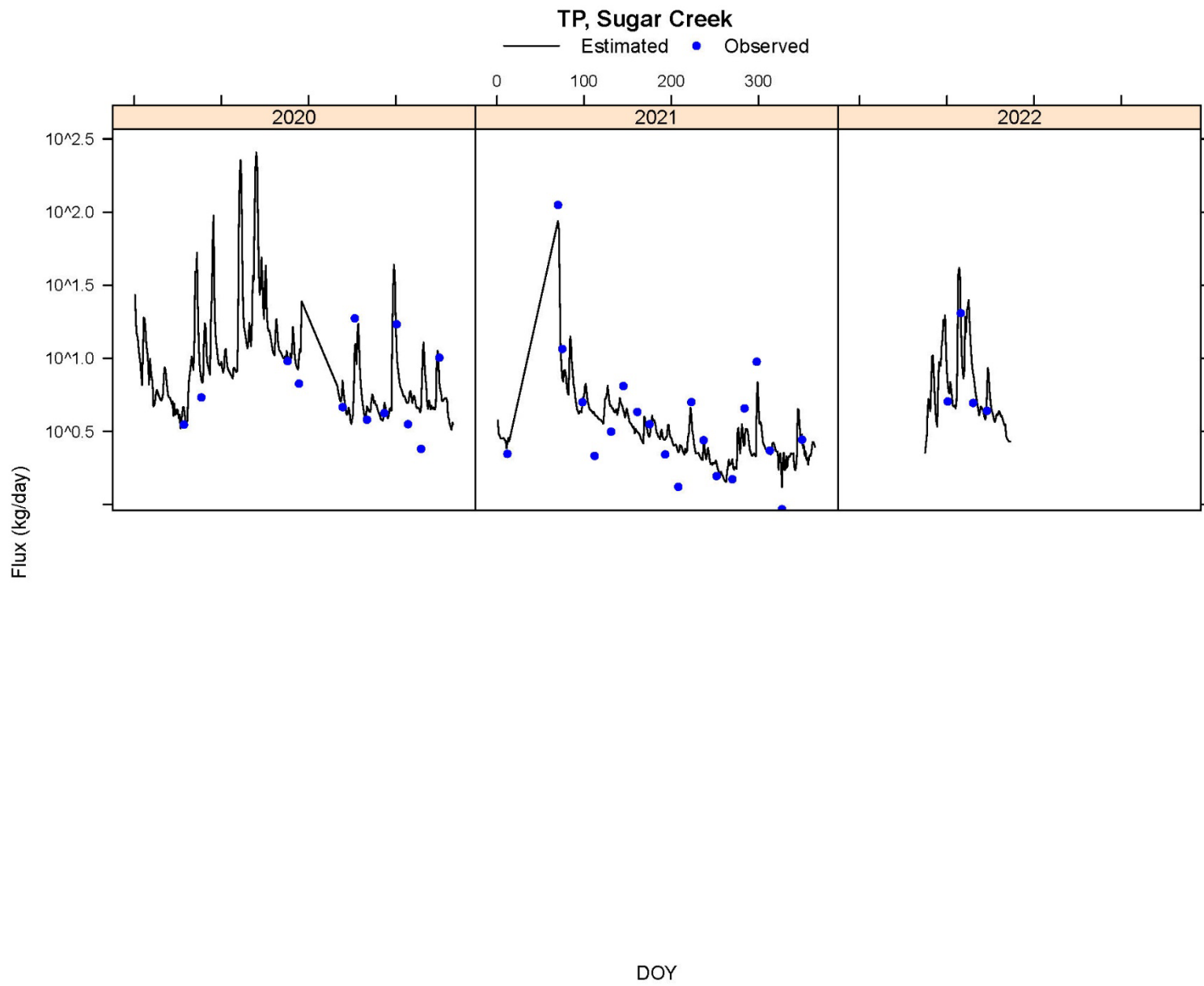


TP, Mukwonago River at Mukwonago

— Estimated • Observed

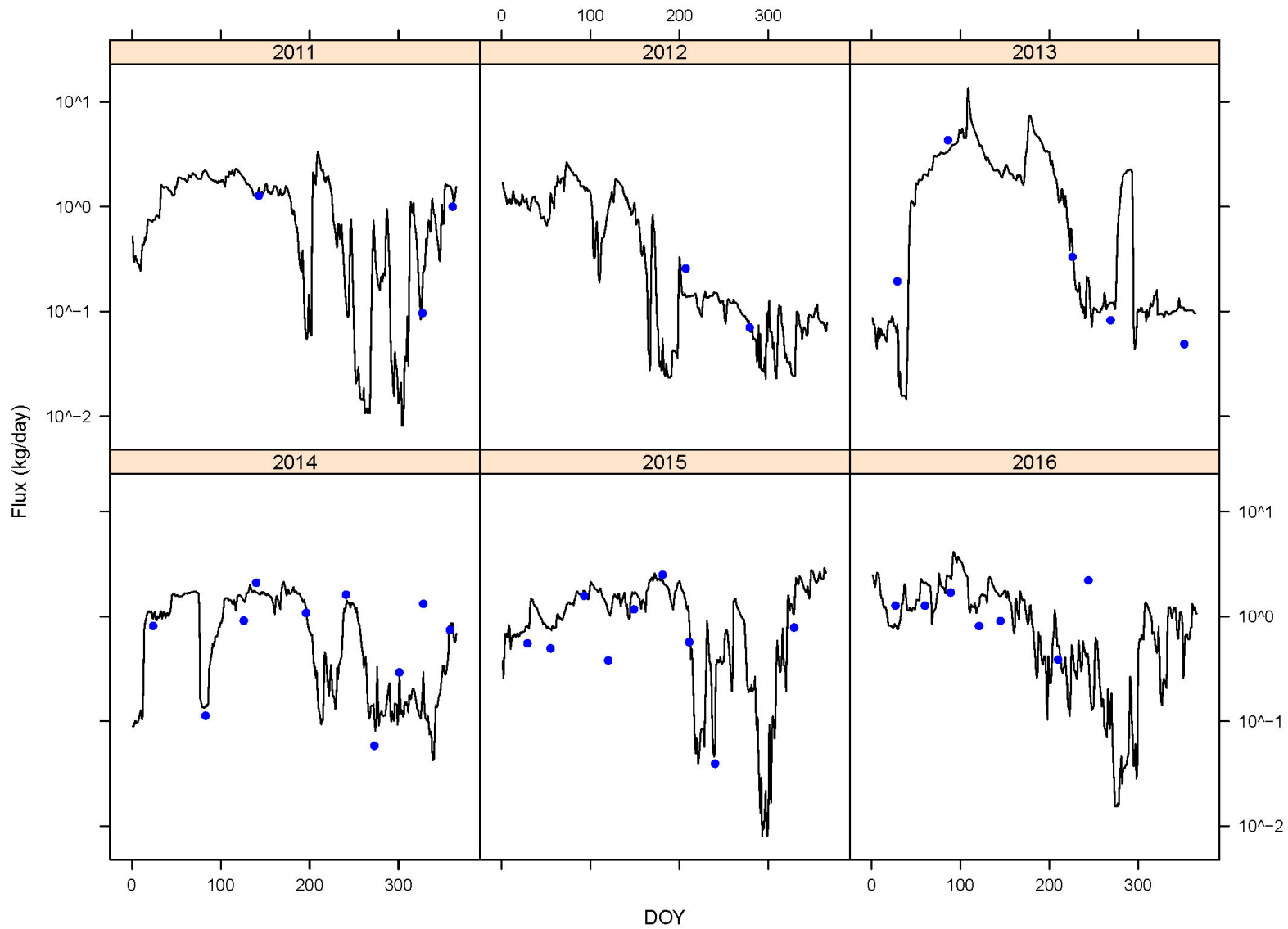






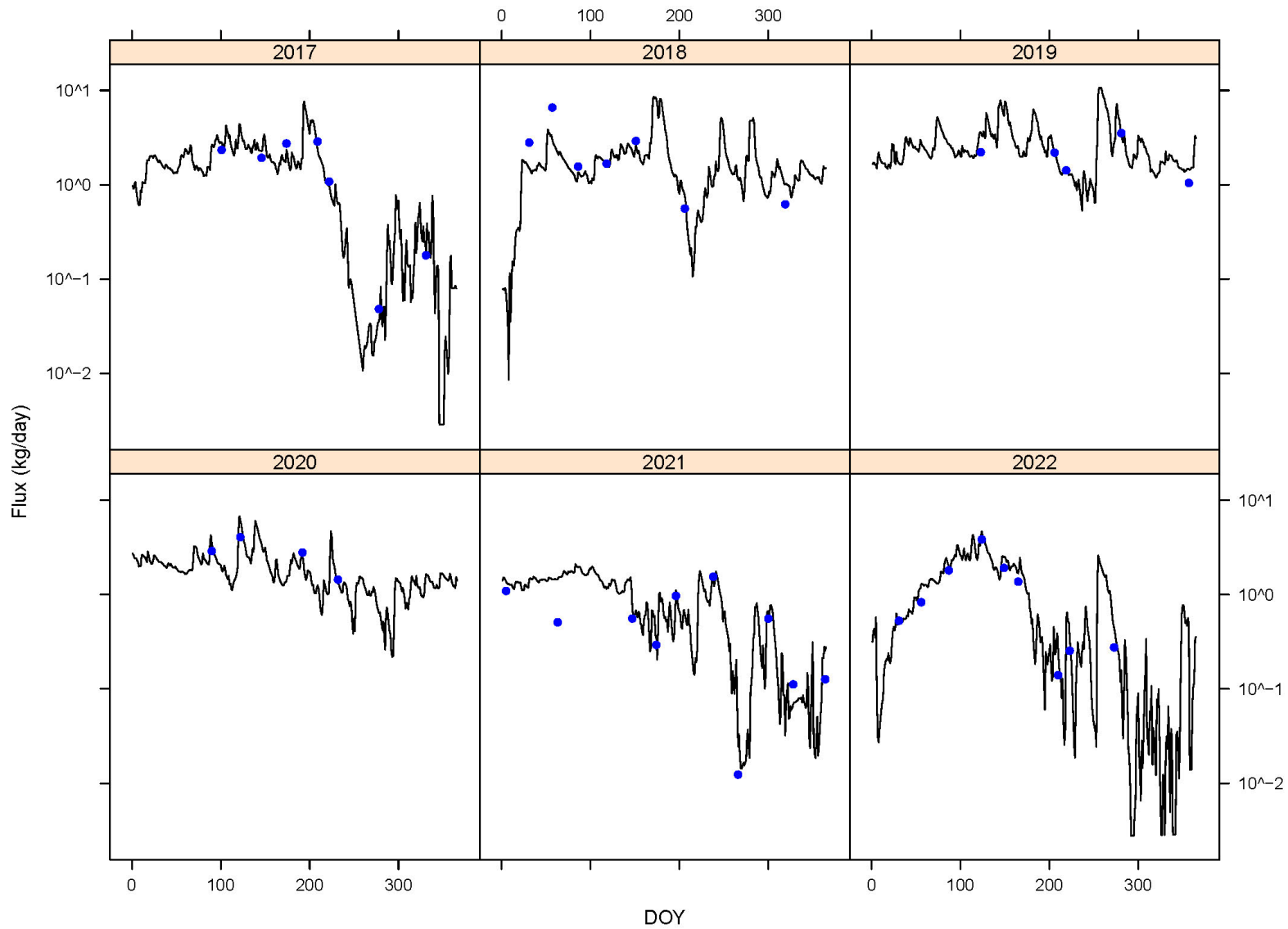
TP, White River at Lake Geneva

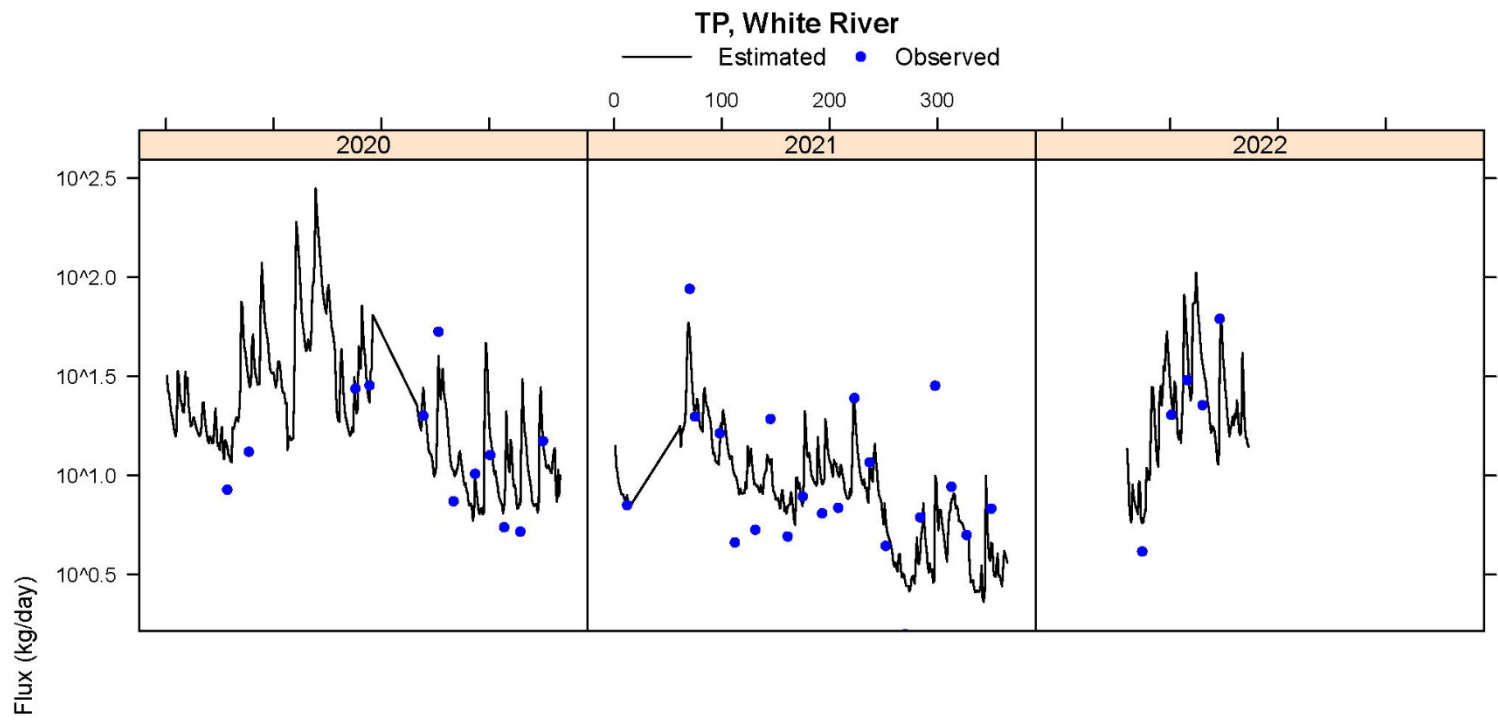
— Estimated • Observed

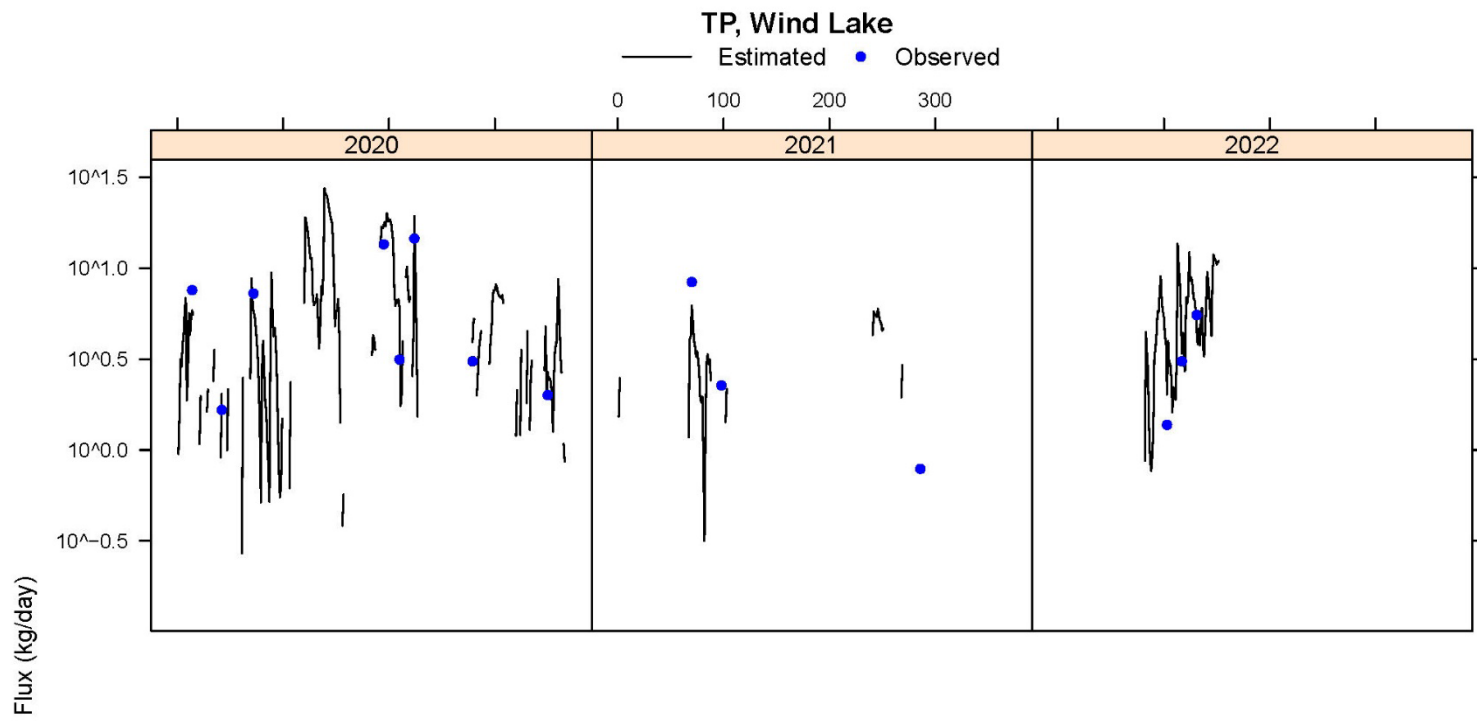


TP, White River at Lake Geneva

— Estimated • Observed







DOY